Electric Potential

*Concepts and Principles*

**An Alternative Approach**

The electric field surrounding electric charges and the magnetic field surrounding moving electric charges can both be conceptualized as information embedded in space. In both cases, the information is embedded as vectors, detailing both the magnitude and direction of each field. Moreover, when this information is “read” by other moving electric charges, the result is a force acting on the charge. These forces can be calculated to allow us to determine the subsequent motion of the charge.

Just as in mechanics, there is an alternative to this force-based approach to analyzing the behavior of electric charges. In this chapter I will define a new field, the electric potential, which surrounds every electric charge. This field differs from the electric field because it is a scalar field, meaning that this field has only a magnitude at every point in space and no associated direction. Moreover, the information in this field, when read by other electric charges, does not result in a force on the charge but rather determines the electric potential energy the charge possesses at that point in space.

Understanding how to calculate this new field, how this field relates to energy, and how this field is related to the electric field will be the focus of this chapter.

**A Gravitational Analogy**

Rather than thinking in terms of the gravitational force and Newton’s Second Law, an alternative way to examine mechanics scenarios is by using the concept of gravitational potential energy and the conservation of energy.

In the force approach, we envision a vector field surrounding the earth, regardless of whether a second mass is nearby to interact with this field. If a mass is present, the mass interacts with this field and feels a gravitational force. This idea is captured in the equation:

\[ \vec{F}_G = m\vec{g} \]
In the energy approach, we can envision a scalar field, the gravitational potential, which is present regardless of whether a second mass is nearby to interact with this field. If a mass is present, the mass interacts with this field and has gravitational potential energy. Near the surface of the earth, the familiar expression for gravitational potential energy:

\[ U_G = mgh \]

can be thought of as the product of the mass of the object and this pre-existing gravitational potential, \( V_G \):

\[ U_G = mV_G \]

if we define the gravitational potential by:

\[ V_G = gh \]

Although we didn’t use the concept of gravitational potential while studying mechanics, it will prove to be a very useful concept in our study of electrical phenomenon. The general expression for gravitational potential, valid regardless of distance from a massive object, is:

\[ V_G = \frac{-Gm}{r} \]

In summary, just as a mass will interact with the vector gravitational field as a force, a mass will interact with the scalar gravitational potential field as potential energy.

The situation is very similar for electrical phenomenon. We can envision a scalar field, the electric potential, which is present regardless of whether a second charge is nearby to interact with this field. If a charge is present, the charge interacts with this field and has electric potential energy. The electric potential, \( V_E \), is defined by the relationship:

\[ V_E = \frac{kq}{r} \]

where

- \( q \) is the source charge, the electric charge that creates the field,
- and \( r \) is the distance between the source charge and the point of interest.

This leads to an expression for the electric potential energy of:

\[ U_E = qV_E \]

where

- \( q \) is the charge on the particle of interest, the charge that is interacting with the field,
- and \( V_E \) is the net electric potential at the location of the particle of interest (created by all of the other charged particles in the universe).
We will typically leave the subscript off the electric potential and electric potential energy unless the possibility of confusion with the gravitational potential and potential energy are present.

### Relating the Electric Field and the Electric Potential

The electric field and the electric potential are not two, independent fields. They are two independent ways of conceptualizing the effect that an electric charge has on the space surrounding it. Just as problems in mechanics can be analyzed using a force-approach or an energy-approach, problems dealing with electrical phenomenon can be analyzed by focusing on the electric field or on the electric potential.

Additionally, just as it is sometimes necessary in mechanics to transfer between force and energy representations, it is sometimes necessary to transfer between the electric field and electric potential representations. The relationship between the two fields can be understood by examining the expression for work, which relates force to transfer of energy.

Utilizing the dot product, the work done in moving a particle from an initial point, i, to a final point, f, can be written as:

\[ W_{i\to f} = \int_i^f \vec{F} \cdot d\vec{l} \]

where \( d\vec{l} \) is an infinitesimal portion of the path along which the particle moves. You may also recall that the difference in potential energy between initial and final locations is defined as the opposite of the work needed to move the particle between the two points:

\[ \Delta U = U_f - U_i = -W_{i\to f} \]

Putting these two ideas together yields:

\[ \Delta U = -\int_i^f \vec{F} \cdot d\vec{l} \]

We can now use this result to relate the electric potential and the electric field. Substituting in expressions for potential energy and force in terms of the fields that convey them leads to:

\[ \Delta(qV) = -\int_i^f (q\vec{E}) \cdot d\vec{l} \]

\[ \Delta V = -\int_i^f \vec{E} \cdot d\vec{l} \]

In English, this final result states that the electric potential difference between any two points is defined as the negative of the integral of the electric field along a path connected the two points. (I’m sure that doesn’t seem like particularly clear English, but this idea will become more tangible once you get to work on some of the fun activities in this chapter.) The bottom line is that the electric potential can be determined by integrating the electric field, and, conversely, the electric field can be determined by differentiating the electric potential.
Electric Potential

Analysis Tools

Point Charges

Find the electric potential at the indicated point. The charges are separated by a distance 4a.

The electric potential at the point specified will be the sum of the electric potential from the left charge \( V_L \) and the electric potential from the right charge \( V_R \).

\[
V = V_L + V_R \\
V_L = \frac{k(2q)}{\sqrt{(3a)^2 + (a)^2}} + \frac{k(-q)}{\sqrt{(a)^2 + (a)^2}} \\
V = \frac{2kq}{\sqrt{10a}} - \frac{kq}{\sqrt{2a}} \\
V = (\frac{2}{\sqrt{10}} - \frac{1}{\sqrt{2}}) \frac{kq}{a} \\
V = -0.0746 \frac{kq}{a}
\]

Notice that since the electric potential is a scalar, calculating the electric potential is often much easier than calculating the electric field.
Continuous Charge Distribution

The plastic rod of length L at right has uniform charge density \( \lambda \). Find the electric potential at all points to the right of the rod on the x-axis.

Since electric charge is discrete, the electric potential can always be calculated by summing the electric potential from each of the electrons and protons that make up an object. However, macroscopic objects contain a lot of electrons and protons, so this summation has many, many terms:

\[
V = \sum_{\text{every electron and proton}} \frac{kq}{r}
\]

As described earlier, we will replace this summation over a very large number of discrete charges with an integral over a hypothetically continuous distribution of charge. This leads to a relationship for the electric potential at a particular point in space, from a continuous distribution of charge, of:

\[
V = \int \frac{k(dq)}{r}
\]

where \( dq \) is the charge on a infinitesimally small portion of the object, and the integral is over the entire physical object.

The steps for finding the electric potential from a continuous distribution of charge are:

1. Carefully identify and label the location of the differential element on a diagram of the situation.
2. Carefully identify and label the location of the point of interest on a diagram of the situation.
3. Write an expression for \( dq \), the charge on the differential element.
4. Write an expression for \( r \), the distance between the differential element and the point of interest.
5. Insert your expressions into the integral for the electric potential.
6. Carefully choose the limits of integration.
7. Evaluate the integral.

I’ll demonstrate below each of these steps for the scenario under investigation.
1. Carefully identify and label the location of the differential element on a diagram of the situation.

The differential element is a small (infinitesimal) piece of the object that we will treat like a point charge. The location of this differential element must be arbitrary, meaning it is not at a "special" location like the top, middle, or bottom of the rod. Its location must be represented by a variable, where this variable is the variable of integration and determines the limits of the integral.

For this example, select the differential element to be located a distance “s” above the center of the rod. The length of this element is “ds”.
(Later, you will select the limits of integration to go from –L/2 to +L/2 to allow this arbitrary element to “cover” the entire rod.)

2. Carefully identify and label the location of the point of interest on a diagram of the situation.

You are interested in the electric potential at all points along the x-axis to the right of the rod. Therefore, select an arbitrary location along the x-axis and label it with its location.

3. Write an expression for \( dq \), the charge on the differential element.

The rod has a uniform charge density \( \lambda \), meaning the amount of charge per unit length along the rod is constant. Since the differential element has a length ds, the total charge on this element (dq) is the product of the density and the length:

\[
dq = \lambda ds
\]

4. Write an expression for \( r \), the distance between the differential element and the point of interest.

By Pythagoras’ theorem, the distance between the differential element and the point of interest is:

\[
r = \sqrt{x^2 + s^2}
\]

6. Insert your expressions into the integral for the electric potential.

\[
V = \int \frac{k(dq)}{r}
\]

\[
V = \int \frac{k\lambda ds}{\sqrt{x^2 + s^2}}
\]
7. Carefully choose the limits of integration.

The limits of integration are determined by the range over which the differential element must be “moved” to cover the entire object. The location of the element must vary between the bottom of the rod (-L/2) and the top of the rod (+L/2) in order to include every part of the rod. The two ends of the rod form the two limits of integration.

\[
V = \int_{-L/2}^{L/2} \frac{k\lambda ds}{\sqrt{x^2 + s^2}}
\]

8. Evaluate the integral.

\[
V = \int_{-L/2}^{L/2} \frac{k\lambda ds}{\sqrt{x^2 + s^2}} = k\lambda \left[ \ln(s + \sqrt{x^2 + s^2}) \right]_{-L/2}^{L/2}
\]

\[
V = k\lambda \left[ \ln\left(\frac{L}{2} + \sqrt{\frac{L^2}{4}}\right) - \ln\left(\frac{-L}{2} + \sqrt{\frac{L^2}{4}}\right) \right]
\]

This expression looks daunting, but its limiting behaviors (as x approaches zero and x approaches infinity) are correct. Consider the case where x gets smaller and smaller, approaching zero:

\[
V = k\lambda \left[ \ln\left(\frac{L}{2} + \sqrt{\left(0\right)^2 + \frac{L^2}{4}}\right) - \ln\left(\frac{-L}{2} + \sqrt{\left(0\right)^2 + \frac{L^2}{4}}\right) \right]
\]

This expression diverges (goes to infinity) as x becomes close to zero, as you should expect since the electric potential directly “on” an electric charge is infinite.
As $x$ gets larger and larger, the $L/2$ terms become insignificant:

\[
V = k\lambda \left[ \ln\left( \frac{L}{2} + \sqrt{x^2 + \frac{L^2}{4}} \right) \right]
\]

\[
V = k\lambda \left[ \ln\left( \frac{L}{2} + \sqrt{x^2} \right) \right]
\]

\[
V = k\lambda \left[ \ln\left( \frac{x}{x} \right) \right]
\]

This expression becomes zero as $x$ becomes infinite, as you should expect since the electric potential extremely far from an electric charge is zero.

---

### Potential Difference

The long, hollow plastic cylinder at right has inner radius $a$, outer radius $2a$, and uniform charge density $\rho$. Find the electric potential difference between the inner and outer radius.

An alternative method for calculating the electric potential at a point, or the electric potential difference between two points, is by using knowledge of the electric field. The following relation,

\[
\Delta V = -\int \vec{E} \cdot d\vec{l}
\]

states that the potential difference between two points can be determined by integrating the component of the electric field that lies along the path connecting the two points. This means that if you know the electric field in a region of space, you can easily (more or less) find the potential difference between any two points that lie in that region. This relation is particularly useful in conjunction with Gauss’ Law for situations with cylindrical or spherical symmetry.
To find the potential difference between points a and b (i.e., what a voltmeter would read if connected across points a and b), we need the electric field in this region. Gauss’ Law can be used to find that (review Gauss’ Law if this step is a little fuzzy):

\[ E = \frac{\rho(r^2 - a^2)}{2\varepsilon_0 r} \]

This electric field points radially away from the center of the cylinder.

For simplicity, choose a path that directly connects a to 2a, i.e., a radial path.

\[ \Delta V = -\int \vec{E} \cdot d\vec{l} \]

\[ V_{2a} - V_a = -\int_a^{2a} \frac{\rho(r^2 - a^2)}{2\varepsilon_0 r} \hat{r} \cdot d\hat{r} \]

\[ V_{2a} - V_a = -\frac{\rho}{2\varepsilon_0} \left[ r - a^2 \ln \frac{r}{a} \right]_a^{2a} \]

\[ V_{2a} - V_a = -\frac{\rho}{2\varepsilon_0} \left[ \frac{1}{2}(2a^2 - a^2) - a^2 \ln \frac{2a}{a} \right] \]

\[ V_{2a} - V_a = -\frac{\rho}{2\varepsilon_0} \left[ \frac{3}{2}a^2 - a^2 \ln 2 \right] \]

\[ V_{2a} - V_a = -\frac{\rho a^2}{2\varepsilon_0} \left[ \frac{3}{2} - \ln 2 \right] \]

\[ V_{2a} - V_a = -0.403 \frac{\rho a^2}{\varepsilon_0} \]

Notice that this method directly calculates the difference in electric potential between two points, without actually determining the value of the electric potential at either point. Since electric potential is related to electric potential energy, this method allows you to find the difference in energy between two points but not the actual value of the energy of an electric charge.

This should strike you as quite similar to the gravitational case. For gravitational potential energy, the choice of the zero-point is arbitrary and only energy differences lead to changes in kinetic energy. For electrical potential energy, the situation is identical. The zero-point of electric potential energy (and electric potential) is typically taken at infinity, although you can “zero” the potential at a more convenient point by “grounding” the system at that point. The physical act of grounding a point on an electrical device is mathematically equivalent to setting the potential equal to zero at that point.
Electric Potential Energy

In many applications, oppositely charged parallel plates (with small holes cut for the beam to pass through) are used to accelerate beams of charged particles. In this example, a proton is injected at $v_i$ into the space between the plates. The potential difference between the plates is $\Delta V$ (the left plate is at higher potential). What is the velocity of the proton as it exits the device?

Since the electric potential changes as the proton moves from the left plate to the right plate, its potential energy changes. This change in potential energy results in a change in kinetic energy of the proton by energy conservation.

\[
K_i + U_i = K_f + U_f
\]
\[
\frac{1}{2}mv_i^2 + qV_{\text{Left}} = \frac{1}{2}mv_f^2 + qV_{\text{Right}}
\]
\[
\frac{1}{2}mv_i^2 + e(V_{\text{Left}} - V_{\text{Right}}) = \frac{1}{2}mv_f^2
\]

\[
v_f = \sqrt{v_i^2 + \frac{2e(\Delta V)}{m}}
\]
Electric Potential

Activities
The two charges below are separated by a distance 4a. Sketch graphs of electric potential along the lines indicated.

a. At $y = 0$

b. At $y = 2a$

c. At $y = 4a$

Electric potential is inversely proportional to the distance from the source. Thus, the potential diverges as the distance from the source goes to zero, which occurs at $x = \pm 2a$, and goes to zero as the distance becomes very large. Between the charges, the potential stays positive and finite.
The two charges below are separated by a distance 4a. Sketch graphs of electric potential along the lines indicated.

a. At x = 0

b. At x = 2a

c. At x = 4a
The two charges below are separated by a distance 4a. Sketch graphs of electric potential along the lines indicated.

a. At $y = 0$

b. At $y = 2a$

c. At $x = 0$

The potential must go to $+\infty$ at $x = -2a$ and $-\infty$ at $x = +2a$, and also go to zero at large distances. At $x = 0$, the potential is positive because of the larger positive charge, but must become negative as you approach the negative charge. The location of the $V = 0$ point is twice as far from the positive charge as it is from the negative charge.
The two charges below are separated by a distance 4a. Sketch graphs of electric potential along the lines indicated.

a. At $x = 0$

b. At $x = 2a$

c. At $y = 2a$
The two charges below are separated by a distance 6a. Sketch graphs of electric potential along the lines indicated.

a. At \( x = 0 \)

![Graph at x = 0](image)

b. At \( y = 0 \)

![Graph at y = 0](image)

c. At \( y = -2a \)

![Graph at y = -2a](image)
The two charges below are located as shown. Each grid square has width \( a \). Sketch graphs of electric potential along the lines indicated.

a. At \( x = 0 \)

b. At \( y = 0 \)

c. At \( y = a \)
For each of the paths described below, rank the paths on the basis of the change in electric potential.

A. from c to d  
B. from d to c  
C. from d to e  
D. from d to b  
E. from e to f  
F. from b to a

Largest Negative

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

The only paths that result in a positive change in potential (i.e., an increase in potential) are B and E. The change along path B is much larger since the potential actually doubles along this path.

Path D results in no change in potential since both points are equidistant from the source charge.

Of the remaining paths that result in a negative change in potential (i.e., a decrease in potential), hopefully it’s clear that path A results in the largest magnitude decrease, since the potential is quite steep close to the charge and the potential is cut in half along this path. Since paths C and F both begin at the same value of potential, path C must have a larger magnitude since it ends farther from the source charge, thus ensuring that the potential has decreased by a larger amount.
For each of the paths described below, rank the paths on the basis of the change in electric potential.

<table>
<thead>
<tr>
<th>Path</th>
<th>Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>from c to d</td>
</tr>
<tr>
<td>B</td>
<td>from d to c</td>
</tr>
<tr>
<td>C</td>
<td>from d to e</td>
</tr>
<tr>
<td>D</td>
<td>from d to b</td>
</tr>
<tr>
<td>E</td>
<td>from e to f</td>
</tr>
<tr>
<td>F</td>
<td>from b to a</td>
</tr>
</tbody>
</table>

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative _____

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
For each of the locations indicated below, rank the locations on the basis of the electric potential.

Largest Positive  1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative  _____

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
For each of the locations indicated below, rank the locations on the basis of the electric potential.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>b</td>
<td>c</td>
<td>d</td>
<td>e</td>
<td>f</td>
<td></td>
</tr>
</tbody>
</table>

**Largest Positive**  
1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____  

**Largest Negative**  
_____  

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
For each of the paths described below, rank the paths on the basis of the change in electric potential.

A from b to a
B from f to e
C from c to d
D from c to e
E from c to b
F from d to a

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative _____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
For each of the paths described below, rank the paths on the basis of the change in electric potential.

A  from b to a
B  from f to e
C  from c to d
D  from c to e
E  from c to b
F  from d to a

Largest Positive  1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____  Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
A region of space is filled with the electric potential illustrated below. Various positions within this region are indicated by the points a through f.

\[\begin{array}{ccc}
\text{A} & \text{an alpha particle (net charge } +2e) \text{ is at rest at a} \\
\text{B} & \text{an electron is at rest at b} \\
\text{C} & \text{a proton is at rest at c} \\
\text{D} & \text{an electron is at rest at d} \\
\text{E} & \text{a neutron is at rest at e} \\
\text{F} & \text{a proton is at rest at f} \\
\end{array}\]

Rank these particles on the basis of their electric potential energy.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative _____

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
A region of space is filled with the electric potential illustrated below. Various positions within this region are indicated by the points a through f. Individual particles are very slowly moved between these positions by an external force.

| A | a proton is moved from a to d |
| B | an electron is moved from d to a |
| C | a proton is moved from d to f |
| D | an electron is moved from b to c |
| E | a neutron is moved from a to e |
| F | an alpha particle (net charge +2e) is moved b to d |

a. Rank these motions on the basis of the change in electric potential energy of the particle during the motion.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative _____

The ranking cannot be determined based on the information provided.

b. Rank these motions on the basis of the work done on the particle by the external force during the motion.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative _____

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
Find the electric potential at the three indicated points along the chain of chloride (Cl⁻) ions and sodium (Na⁺) ions. The interionic separation is $2.82 \times 10^{-10}$ m.

Mathematical Analysis
Find the electric potential at the three indicated points in the array of sodium ions (Na⁺) and chloride ions (Cl⁻). The interionic separation is $2.82 \times 10^{-10}$ m.

Mathematical Analysis
Find the electric potential at each of the indicated points. The charges are separated by a distance $4a$.

Mathematical Analysis
Find the electric potential at each of the indicated points. The charges are separated by a distance 4a.

Mathematical Analysis
The two charges at right are separated by a distance $2a$. Find the electric potential at all points on the $x$-axis and sketch it below.

**Mathematical Analysis**

*for $x > a$:*

*for $-a < x < a$:*

*for $x < -a$:*
The two charges at right are separated by a distance 2a. Find the electric potential at all points on the y-axis and sketch it below.

Mathematical Analysis
The plastic rod at right has charge +Q uniformly distributed along its length L. Find the electric potential at all points on the x-axis to the right of the rod.

Mathematical Analysis

- Carefully identify and label “s” the location of the differential element on the diagram above.
- Carefully identify and label “x” the location of the point of interest on the diagram above.
- Write an expression for dq, the charge on the differential element.

- Write an expression for r, the distance between the differential element and the point of interest.

- Write an integral expression for the electric potential. Choosing limits of integration, calculate the integral.

Questions

At x = L/2, what should V equal? Does your function agree with this observation?

If x >> L, what should V equal? Does your function agree with this observation?
The plastic rod at right has charge \(-Q\) uniformly distributed along its length \(L\). Find the electric potential at all points on the \(y\)-axis above the rod.

**Mathematical Analysis**
- Carefully identify and label "\(s\)" the location of the differential element on the diagram above.
- Carefully identify and label "\(y\)" the location of the point of interest on the diagram above.
- Write an expression for \(dq\), the charge on the differential element.
- Write an expression for \(r\), the distance between the differential element and the point of interest.
- Write an integral expression for the electric potential. Choosing limits of integration, calculate the integral.

**Questions**
At \(y = 0\), what should \(V\) equal? Does your function agree with this observation?

If \(y \gg L\), what should \(V\) equal? Does your function agree with this observation?
The plastic rod at right forms a quarter-circle of radius $R$ and has uniform charge density $\lambda$. Find the electric potential at the origin.

**Mathematical Analysis**

- Carefully identify and label "$\theta$" the location of the differential element on the diagram above.
- Carefully identify and label the location of the point of interest on the diagram above.
- Write an expression for $dq$, the charge on the differential element.

- Write an expression for $r$, the distance between the differential element and the point of interest.

- Write an integral expression for the electric potential. Choosing limits of integration, calculate the integral.
The plastic rod at right forms a half-circle and has charge \(-Q\) uniformly distributed along its length \(L\). Find the electric potential at the origin.

**Mathematical Analysis**

- Carefully identify and label “\(\theta\)” the location of the differential element on the diagram above.
- Carefully identify and label the location of the point of interest on the diagram above.
- Write an expression for \(dq\), the charge on the differential element.

- Write an expression for \(r\), the distance between the differential element and the point of interest.

- Write an integral expression for the electric potential. Choosing limits of integration, calculate the integral.
The plastic rod at right forms a circle of radius \( R \) and has charge \( Q \) uniformly distributed along its length. Find the electric potential at the origin.

Mathematical Analysis

- Carefully identify and label \( \theta \) the location of the differential element on the diagram above.
- Carefully identify and label the location of the point of interest on the diagram above.
- Write an expression for \( dq \), the charge on the differential element.

- Write an expression for \( r \), the distance between the differential element and the point of interest.

- Write an integral expression for the electric potential. Choosing limits of integration, calculate the integral.
The solid plastic sphere at right has radius $R$ and charge $+Q$ uniformly distributed throughout its volume. Find the electric potential at all points in space and sketch it below, labeling the value of the potential at $r = R$.

**Mathematical Analysis**

For $r > R$:

For $r < R$:

\[ V \]

\[ r \]

\[ R \]

\[ r \]
The solid metal sphere at right has radius $R$ and total charge $Q$. Find the electric potential at all points in space and sketch it below, labeling the value of the potential at $r = R$.

**Mathematical Analysis**

*for* $r > R$:

*for* $r < R$:
The hollow plastic sphere at right has inner radius $a$, outer radius $b$, and uniform charge density $\rho$. Find the electric potential difference between $a$ and $b$.

Mathematical Analysis
The hollow metal sphere at right has inner radius $a$, outer radius $b$, and total charge $+Q$. Find the electric potential at the center of the sphere.

Mathematical Analysis
A hollow metal sphere of radius 3.0 cm is charged to 20 kV relative to ground. What is the total charge and charge density on the sphere?

Mathematical Analysis
The inner wire of the very long coaxial cable at right has a diameter of 1.5 mm and the outer shell a diameter 1.5 cm. If the linear charge density on the inner wire is 2.3 nC/m, what potential difference was applied between the wire and the shell?

Mathematical Analysis
A Geiger counter consists of a thin inner wire and a concentric cylindrical conducting shell. The space between is typically filled with an inert gas, but in this example it is filled with air. The wire is charged such that the potential difference between the wire and shell is very near 3 kV/mm, the breakdown potential for air. If a charged particle enters the device, its energy will cause the air to electrically breakdown, sending a spark between wire and shell. This is the distinctive “click” of the counter. If the inner wire has a diameter of 50 μm and the outer shell a diameter 2.5 cm, find the linear charge density on the inner wire.

Mathematical Analysis
A sodium ion (Na⁺) and chloride ion (Cl⁻) are separated by $2.82 \times 10^{-10}$ m.

Mathematical Analysis

a. Find the electric potential energy of the sodium ion.

b. Find the electric potential energy of the chloride ion.

c. How much work would need to be done to break the bond between the ions?

d. Assume the chloride ion was replaced with a second sodium ion. Find the new electric potential energy of either sodium ion.

e. Explain how the algebraic sign of the electric potential energy can be used to determine if a charge configuration is bound or unbound.
Find the electric potential energy of each ion. The interionic separation is $2.82 \times 10^{-10}$ m.

Mathematical Analysis
Find the electric potential energy of each ion. The interionic separation is $2.82 \times 10^{-10} \text{m}$.

Mathematical Analysis
In alpha decay, an alpha particle (a bound state of 2 protons and 2 neutrons) escapes from a heavy nucleus and is propelled away due to electric repulsion. For example, a radon nucleus (atomic number 86) will spontaneously transform into a polonium nucleus (atomic number 84) and an alpha particle. Immediately following this transformation, both the polonium nucleus and the alpha particle can be assumed to be at rest, and separated by approximately $50 \times 10^{-15}$ m.

**Mathematical Analysis**

*a. What is the initial electric potential energy of the alpha particle?*

*b. What is the velocity of the alpha particle when it is far from the polonium nucleus? Since the polonium is much more massive than the alpha, assume the polonium stays approximately at rest.*

*c. How far from the polonium nucleus is the alpha particle when it achieves 90% of the velocity calculated in (b)?*
In an attempt to achieve nuclear fusion, a proton is fired at a fixed deuterium nucleus. (Deuterium is an isotope of hydrogen with one proton and one neutron in the nucleus.) If the distance of closest approach between the proton and the deuterium nucleus is about $1.0 \times 10^{-15}$ m, the proton and deuterium have a good chance of fusing.

**Mathematical Analysis**

a. With what speed must the proton be fired in order to have a good chance of fusing with the deuterium? Assume the proton travels directly toward the deuterium and momentarily “stops” at the distance of closest approach.

b. If deuterium was launched at a fixed target of protons, rather than vice-versa, what launch velocity would be needed for the deuterium and protons to undergo fusion?

c. If the proton is launched with only one-half the speed calculated in (a), how close will it get to the deuterium nucleus?
The two particles at right, with the same algebraic sign of charge, are held at rest with separation \( r_0 \). The particles are then released, and at a later time have a separation, \( r \), and velocities, \( v_1 \) and \( v_2 \).

**Mathematical Analysis**

a. Apply conservation of energy to create a relationship between \( v_1 \) and \( v_2 \).

b. Apply conservation of momentum to create a relationship between \( v_1 \) and \( v_2 \).

c. Eliminate \( v_2 \) from your system of two equations and determine \( v_1 \) as a function of the parameters defined above.

c. Determine the maximum speed of two protons released from initial separation 1.0 nm.
In the Bohr model of the hydrogen atom, the ground state of hydrogen consists of a proton encircled by an electron at radius \( a_0 \). All allowed orbits exist at radii given by \( n^2 a_0 \), where \( n \) is the orbit number (1, 2, 3, etc.).

**Mathematical Analysis**

a. Determine the total energy of the electron as a function of the electrostatic constant \( (k) \), the elementary charge \( (e) \), the orbit number \( (n) \), and the ground state radius \( (a_0) \). (Hint: You will need to apply Newton’s Second Law to the circular motion of the electron to determine its velocity (and kinetic energy).)

b. With \( a_0 = 5.29 \times 10^{-11} \text{ m} \), numerically evaluate your expression in (a), leaving only the orbit number \( (n) \) unspecified.

c. Determine the energy needed to excite the electron from the ground state to the first excited state.

d. Determine the energy needed to ionize hydrogen.
In many applications, oppositely charged parallel plates (with small holes cut for the beam to pass through) are used to accelerate beams of charged particles. In this example, an electron is injected at $2.0 \times 10^7$ m/s into the space between the plates. The plates are 5 cm apart.

Mathematical Analysis

a. If the electron exits the device at $2.5 \times 10^7$ m/s, what is the potential difference between the plates?

b. If a proton entered the same device at the same speed, with what speed would it exit the device?

c. What is the electric field between the plates?
In many applications, oppositely charged parallel plates (with small holes cut for the beam to pass through) are used to accelerate beams of charged particles. In this example, a proton is injected at $2.0 \times 10^6$ m/s into the space between the plates. The potential difference between the plates is 20 kV (the left plate is at higher potential). The plates are 5 cm apart.

Mathematical Analysis

a. What is the speed of the proton as it exits the device?

b. If the separation between the plates is increased to 10 cm while the potential difference is held constant at 20 kV, what is the exit speed of the proton?

c. What happened to the charge density on the plates as the separation was doubled while holding the potential difference constant?
A proton is accelerated from rest through a potential difference $\Delta V$ and then enters a region of uniform magnetic field $B$.

**Mathematical Analysis**

a. Determine the radius of the proton’s path ($R$) as a function of the charge ($e$) and mass ($m$) of a proton, $\Delta V$, and $B$.

b. Determine the period of the proton’s orbit ($T$) as a function of the charge ($e$) and mass ($m$) of a proton, $\Delta V$, and $B$.

c. If the accelerating potential is doubled, what happens to the orbital radius and period?
An electron is accelerated from rest through a potential difference $\Delta V$ and then enters a region of uniform magnetic field $B$. A target screen is a distance $D$ from the exit of the accelerator.

**Mathematical Analysis**

a. Determine the minimum accelerating potential ($\Delta V$) needed for the electron to strike the screen as a function of the charge $e$ and mass $m$ of an electron, $D$, and $B$.

b. Evaluate your expression for $D = 10 \text{ cm}$ and $B = 0.5 \text{ T}$.

c. If the electron is accelerated with twice the potential calculated in (b), where on the screen does it strike?