Spiral Physics

Model Two

The constant-force particle model
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Model Specifics

For our second pass through the study of mechanics, we will eliminate one of the major restrictions in our original model. We will now allow objects to move through three-dimensional space. We will still, however, restrict our model with the following approximations.

- The object is acted on by constant forces.
- The object’s size and shape are unimportant.
- The object is classical.
Kinematics

Concepts and Principles

An empirical fact about nature is that motion in one direction (for example, the horizontal) does not appear to influence aspects of the motion in a perpendicular direction (the vertical). Imagine a coin dropped from shoulder height. The elapsed time for the coin to hit the ground, the rate at which its vertical position is changing, and its vertical acceleration are the same whether you do this experiment in a stationary bus or one traveling down a smooth, level highway at 65 mph. The horizontal motion of the coin does not affect these aspects of its vertical motion.\(^1\)

Thus, to completely describe the motion of an object moving both horizontally and vertically, you can first ignore the horizontal motion, and describe only the vertical component of the motion, and then ignore the vertical motion, and describe the horizontal component. Putting these kinematic components together gives you a complete description of the motion. This experimental fact about nature will make analyzing multi-dimensional motion no more conceptually difficult than analyzing one-dimensional motion.

Given this independence between motions in perpendicular directions, the same kinematic concepts and relationships utilized in one-dimensional motion will be utilized for multi-dimensional motion.

Position

The position of an object is its location relative to a well-defined coordinate system. In multi-dimensional situations, however, you must designate coordinate systems for all perpendicular directions of interest. The zero and positive direction for one dimension is completely independent of the zero and positive direction for another direction. The location at which all coordinate system zeros intersect is referred to as the origin of the coordinate system.

Velocity

The velocity is the rate at which the position is changing. Thus, we will define the velocity component in the vertical direction, for example, as the rate at which the vertical position is changing. The velocity component in the vertical direction is completely independent of the horizontal position or the rate at which the horizontal position changes.

\(^1\) Actually, at extremely high speeds the horizontal and vertical motions are not independent. At speeds comparable to the speed of light, the interdependence between horizontal and vertical motion (because of time dilation) becomes noticeable.
As long as the coordinate directions are perpendicular, the speed, or magnitude of the object’s velocity, can be determined by:

\[ v = \sqrt{v_x^2 + v_y^2 + v_z^2} \]

The direction of the object’s velocity can be determined via right-angle trigonometry.

### Acceleration

The acceleration is the rate at which the velocity is changing. Thus, we will define the acceleration component in the vertical direction, for example, as the rate at which the velocity component in the vertical direction is changing. The acceleration component in the vertical direction is *completely* independent of the velocity component in the horizontal direction or the rate at which the velocity component in the horizontal direction changes.

As long as the coordinate directions are perpendicular, the magnitude of the object’s acceleration can always be determined by:

\[ a = \sqrt{a_x^2 + a_y^2 + a_z^2} \]

The direction of the object’s acceleration can be determined via right-angle trigonometry.

Doing kinematics in multiple dimensions involves a concerted effort on your part to disregard motion in one direction when considering motion in a perpendicular direction. The ability to mentally break down a complicated motion into its component motions requires considerable practice.
Kinematics

Analysis Tools

Drawing Motion Diagrams

Motion diagrams are of crucial importance in investigating scenarios involving multi-dimensional motion. For example, analyze the following scenario:

In the shot put, a large mass is thrown at an angle of 22° above horizontal, from a position of 2 m above the ground, a horizontal distance of 25 m.

A motion diagram for this scenario is sketched below.

- Horizontal (x) and vertical (y) coordinate systems are clearly indicated.
- The acceleration is determined by the same method as in one-dimensional motion. In this case, the acceleration was determined near the beginning of the motion. Determining the acceleration at any other time will also indicate that its direction is straight downward.
- In constructing the motion diagram, only a portion of the entire motion of the shot put is illustrated. For this motion diagram, analysis will begin the instant after the shot put leaves the putter’s hand, and analysis will end the instant before the shot put hits the ground. It is of extreme importance to clearly understand the beginning and the end of the motion that you will describe. The acceleration of the shot put while in the putter’s hand, and the acceleration upon contact with the ground, has been conveniently left out of this analysis. Unless explicit information is either provided or desired about these accelerations, it is best to focus analysis on the simplest essentials of the motion.
Drawing Motion Graphs

Let’s look at the situation again:

In the shot put, a large mass is thrown at an angle of 22° above horizontal, from a position of 2 m above the ground, a horizontal distance of 25 m.

The verbal representation of the situation has already been translated into a pictorial representation, the motion diagram. A careful reading of the motion diagram allows the construction of the motion graphs.

**Drawing the position vs. time graph**
First, examine the position of the shot put as it moves through the air. Remember, the analysis of the horizontal position must be independent of the analysis of the vertical position.

**Horizontal Position**
From the motion diagram, the shot put starts at position zero, and then has positive, increasing positions throughout the remainder of its motion. The horizontal position increases by even amounts in even time intervals.
**Vertical Position**
The shot put starts at position zero, increases its vertical position at a rate that is decreasing, then begins to decrease its vertical position at a rate that is increasing, even as it drops to negative positions.

Typically, both the horizontal and vertical positions are displayed on the same axis.

**Drawing the velocity vs. time graph**
In the horizontal direction, the rate at which the position changes is constant. Hence, the horizontal component of velocity is constant, and positive. In the vertical direction, the velocity component begins positive, decreases to zero, and then increases in the negative direction.
**Drawing the acceleration vs. time graph**

From the motion diagram, the acceleration of the shot-put can be determined to be directed downward at every point. Thus, the horizontal component of acceleration is zero and the vertical component is negative, and approximately constant due to our model's approximations.

![Acceleration vs. time graph](image)

**Tabulating Motion Information**

In the shot, a large mass is thrown at an angle of 22° above horizontal, from a position of 2 m above the ground, a horizontal distance of 25 m.

![Motion diagram](image)

Now that you have constructed a motion diagram and motion graphs, you should be able to assign numerical values to several of the kinematic variables. A glance at the situation description should indicate that information is presented about the shot put at two distinct events: when the shot put leaves the putter’s hand and when the shot put strikes the ground. Other information can also be determined about these events by referencing the motion diagram. To organize this information, you should construct a motion table.
Event 1: The instant after the shot put leaves the hand.

\begin{align*}
  t_1 &= 0 \text{ s} \\
  r_{1x} &= 0 \text{ m} \\
  r_{1y} &= 0 \text{ m} \\
  v_{1x} &= v_1 \cos 22^\circ \\
  v_{1y} &= v_1 \sin 22^\circ \\
  a_{1x} &= 0 \text{ m/s}^2 \\
  a_{1y} &= -9.8 \text{ m/s}^2
\end{align*}

Event 2: The instant before the shot put hits the ground.

\begin{align*}
  t_2 &= \\
  r_{2x} &= +25 \text{ m} \\
  r_{2y} &= -2 \text{ m} \\
  v_{2x} &= \\
  v_{2y} &=
\end{align*}

In addition to the information explicitly given (the initial and final positions), information is available about both the initial velocity and the acceleration.

**Initial velocity:** Although the magnitude of the initial velocity \((v_1)\) is unknown, its orientation in space is known. Thus, via the right-angle trigonometry shown below, the components of this unknown magnitude velocity in the horizontal and vertical directions can be determined. Since we will analyze the x- and y-motion separately, we must break the initial velocity into its x- and y-components.

\[
\begin{align*}
  v_{1x} &= v_1 \cos \theta \\
  v_{1y} &= v_1 \sin \theta
\end{align*}
\]

**Acceleration:** The only force acting on the shot-put during the time interval of interest is the force of gravity, which acts directly downward. This is because the analysis of the motion is restricted to the time interval after leaving the thrower’s hand and before striking the ground. Thus, there is no horizontal acceleration of the shot-put and the vertical acceleration has a magnitude of 9.8 m/s².
Doing the Math

In Model 1, you were presented with two kinematic relationships. These relationships are valid in both the horizontal and vertical directions. Thus, you have a total of four relationships with which to analyze the scenario given. In the example above, there are four unknown kinematic variables. You should remember from algebra that four equations are sufficient to calculate four unknowns. Thus, by applying the kinematic relations in both the horizontal and vertical directions, you should be able to determine the initial velocity of the shot-put, the time in the air, and the final horizontal and vertical velocity components.

First, let’s examine the horizontal component of the motion. Note that the positions, velocities, and accelerations in the following equations are all horizontal components.

**x-direction**

\[
\begin{align*}
v_2 &= v_1 + a_{1x}(t_2 - t_1) \\
v_{2x} &= (v_1 \cos 22) + 0(t_2 - 0) \\
v_{2x} &= 0.927v_1 \\
r_2 &= r_1 + v_1(t_2 - t_1) + \frac{1}{2}a_{1x}(t_2 - t_1)^2 \\
25 &= 0 + (v_1 \cos 22)(t_2 - 0) + \frac{1}{2}(0)(t_2 - 0)^2 \\
25 &= 0.927v_1t_2 \\
v_1 &= \frac{27.0}{t_2}
\end{align*}
\]

Now let’s examine the vertical component of the motion. All the positions, velocities, and accelerations in the following equations are now vertical components.

**y-direction**

\[
\begin{align*}
v_2 &= v_1 + a_{1y}(t_2 - t_1) \\
v_{2y} &= (v_1 \sin 22) + (-9.8)(t_2 - 0) \\
v_{2y} &= 0.375v_1 - 9.8t_2 \\
r_2 &= r_1 + v_1(t_2 - t_1) + \frac{1}{2}a_{1y}(t_2 - t_1)^2 \\
-2 &= 0 + (v_1 \sin 22)(t_2 - 0) + \frac{1}{2}(-9.8)(t_2 - 0)^2 \\
-2 &= 0.375v_1t_2 - 4.9t_2^2 \\
-2 &= 0.375(\frac{27.0}{t_2})t_2 - 4.9t_2^2 \\
-2 &= 10.1 - 4.9t_2^2 \\
12.1 &= -4.9t_2^2 \\
2.47 &= t_2^2 \\
t_2 &= 1.57s
\end{align*}
\]

Substituting the value of \( v_1 \) from above yields:

\[
\begin{align*}
-2 &= 0.375(\frac{27.0}{t_2})t_2 - 4.9t_2^2 \\
-2 &= 10.1 - 4.9t_2^2 \\
12.1 &= -4.9t_2^2 \\
2.47 &= t_2^2 \\
t_2 &= 1.57s
\end{align*}
\]

Plugging \( t_2 = 1.57 \) s into all of the remaining equations gives:

\[
\begin{align*}
v_1 &= 17.2 \text{ m/s} \\
v_{2x} &= 15.9 \text{ m/s} \\
v_{2y} &= -8.94 \text{ m/s}
\end{align*}
\]
Kinematics

Hints and Suggestions

Selecting Events

Let’s look again at the shot-putter.

In the shot put, a large mass is thrown at an angle of $22^o$ above horizontal, from a position of 2 m above the ground, a horizontal distance of 25 m.

Imagine a videotape of the shot put event. Fast-forward over the frames showing the shot putter picking up the shot and stepping into the ring. Begin to watch the imaginary videotape frame-by-frame as the shot putter begins to push the shot off of her shoulder and forward. Stop the videotape on the frame when the shot first leaves the putter’s hand.

Why is it so important that we begin the analysis at this frame and explicitly disregard all the motion that has taken place before this frame? The reason is that in every frame preceding this frame, the shot put was in contact with the putter. Thus, the putter was exerting a force on the shot. Since no information is presented concerning this force, we have no way to determine the acceleration during these frames and hence no way to determine any other kinematic variables. Thus, we disregard all motion preceding the instant the shot leaves the putter’s hand because that portion of the motion is simply impossible to analyze with the information provided. Once the shot leaves her hand, the only force acting on the shot is the force of gravity, which greatly simplifies the analysis.

Continue playing the imaginary videotape forward. Begin playing the tape frame-by-frame as the shot approaches the ground. Stop the videotape the frame before the shot hits the ground. We will stop our analysis at this frame. Why? Because starting with the next frame, the shot is in contact with the ground. Once in contact with the ground, an additional, unknown magnitude force begins to act on the shot. Once an unknown magnitude force begins to act, the acceleration of the shot becomes unknown and we are stuck. Thus, we conveniently stop our analysis before things get too complicated!

Since our analysis stops the instant before contact, note that the shot is still moving at this instant. (If it wasn’t, how could it ever reach the ground?) Thus, resist the temptation to think that the velocity of the shot is zero at the end of analysis. The velocity of the shot is ultimately equal to zero (after it makes a big divot into the ground) but that happens long after it strikes the ground and hence long after our analysis is finished.
Kinematics

Activities
For each of the motion diagrams below, determine the algebraic sign (+, - or zero) of the x- and y-position, velocity, and acceleration of the object at location of the three open circles.

a.

b.
For each of the motion diagrams below, determine the algebraic sign (+, - or zero) of the x- and y-position, velocity, and acceleration of the object at location of the three open circles.

a.

b.
Construct the missing motion graphs and/or motion diagram.

a. Motion Graphs

Motion Diagram

- Positive slope on the x-position graph means positive x-velocity.
- Begins at zero x-position and positive y-position.
- Travels at constant speed, positive in x and negative in y.

b. Motion Diagram

- Y-position equals zero when x-position is negative. Later, x-position is zero when y-position is negative.
- Begins at large negative x-position and smaller magnitude positive y-position.
- Positive constant x-velocity and negative constant y-velocity.

Motion Graphs
Construct the missing motion graphs and/or motion diagram.

a. Motion Graphs

Motion Diagram

b. Motion Diagram

Motion Graphs
Construct the missing motion graphs and/or motion diagram.

a. **Motion Graphs**

   ![Motion Graphs](image)

b. **Motion Diagram**

   ![Motion Diagram](image)

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Construct the missing motion graphs and/or motion diagram.

a. **Motion Graphs**

![Motion Graphs](image)

b. **Motion Diagram**

![Motion Diagram](image)

**Motion Graphs**

![Motion Graphs](image)
An object’s motion is represented by the position vs. time graph below. Both the x- and y-position components are indicated on the graph.

a. Rank the object’s distance from the origin at the lettered times.


_____ The ranking cannot be determined based on the information provided.

Since distance is given by Pythagoras’ Theorem, \( D = \sqrt{r_x^2 + r_y^2} \), and \( r_y \) is constant, the distance from the origin is proportional to the magnitude of the x-position.

b. Rank the object’s speed at the lettered times.

Largest 1. ABCDE 2. ___ 3. ___ 4. ___ 5. ___ Smallest

_____ The ranking cannot be determined based on the information provided.

The object moves with constant speed in the positive x-direction.

c. Rank the angle between the object’s velocity and the x-axis at the lettered times, measuring all angles counterclockwise from +x. (Thus, the +y axis is at 90°.)

Largest 1. ABCDE 2. ___ 3. ___ 4. ___ 5. ___ Smallest

_____ The ranking cannot be determined based on the information provided.

Since the object moves with constant speed in the positive x-direction, the angle of its velocity vector is 0°.
An object’s motion is represented by the velocity vs. time graph below. Both the x- and y-velocity components are indicated on the graph.

a. Rank the object’s distance from the origin at the lettered times.

Largest 1. 2. 3. 4. 5. Smallest

The ranking cannot be determined based on the information provided.

Since a velocity graph doesn’t specify the location of the coordinate system, you can’t determine the distance from the origin of the coordinate system.

b. Rank the object’s speed at the lettered times.

Largest 1. 2. 3. 4. 5. Smallest

The ranking cannot be determined based on the information provided.

Since speed is given by \( v = \sqrt{v_x^2 + v_y^2} \), and \( v_y \) is constant, the speed is proportional to the magnitude of the x-position velocity.

c. Rank the angle between the object’s velocity and the x-axis at the lettered times, measuring all angles counterclockwise from +x. (Thus, the +y axis is at 90°.)

Largest 1. 2. 3. 4. 5. Smallest

The ranking cannot be determined based on the information provided.

Draw a motion diagram! The y-velocity is constant and positive, so all of the vectors are in the first and second quadrant. A and B are at > 90°, C is at 90°, and D and E are at < 90°.
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c. Rank the angle between the object’s velocity and the x-axis at the lettered times, measuring all angles counterclockwise from +x. (Thus, the +y axis is at 90°.)
   Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Smallest
   _____ The ranking cannot be determined based on the information provided.
Below are six identical baseballs thrown horizontally at different speeds from different heights above the ground. Assume the effects of air resistance are negligible.

a. Rank these baseballs on the basis of the elapsed time before they hit the ground.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

b. Rank these baseballs on the basis of the magnitude of their vertical velocity when they hit the ground.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
Below are six balls of different mass thrown horizontally at different speeds from the same height above the ground. Assume the effects of air resistance are negligible.

a. Rank these baseballs on the basis of the elapsed time before they hit the ground.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

b. Rank these baseballs on the basis of the horizontal distance traveled before they hit the ground.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
Below are six different directions and speeds at which a baseball can be thrown. In all cases the baseball is thrown at the same height, $H$, above the ground. Assume the effects of air resistance are negligible.

<table>
<thead>
<tr>
<th>V</th>
<th>θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>30 m/s</td>
</tr>
<tr>
<td>B</td>
<td>45 m/s</td>
</tr>
<tr>
<td>C</td>
<td>30 m/s</td>
</tr>
<tr>
<td>D</td>
<td>15 m/s</td>
</tr>
<tr>
<td>E</td>
<td>20 m/s</td>
</tr>
<tr>
<td>F</td>
<td>15 m/s</td>
</tr>
</tbody>
</table>

a. Rank these baseballs on the basis of the maximum height the baseball reaches above the ground.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest _____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

b. Rank these baseballs on the basis of the elapsed time before they hit the ground.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest _____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
At a circus, a human cannonball is shot from a cannon at 15 m/s at an angle of 40° above horizontal. She leaves the cannon 1.0 m off the ground and lands in a net 2.0 m off the ground.

**Motion Diagram**

**Motion Information**

<table>
<thead>
<tr>
<th>Event 1</th>
<th>Event 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_1 = )</td>
<td>( t_2 = )</td>
</tr>
<tr>
<td>( r_{1x} = )</td>
<td>( r_{2x} = )</td>
</tr>
<tr>
<td>( r_{1y} = )</td>
<td>( r_{2y} = )</td>
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<td>( v_{1x} = )</td>
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<td>( v_{1y} = )</td>
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<tr>
<td>( a_{12x} = )</td>
<td>( a_{12y} = )</td>
</tr>
</tbody>
</table>

**Mathematical Analysis**
At the buzzer, a basketball player shoots a desperation shot. She is 10 m from the basket and the ball leaves her hands exactly 1.2 m below the rim. She shoots at 35° above the horizontal and the ball goes in!

**Motion Diagram**

**Motion Information**

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<td>$a_{1x} =$</td>
<td>$v_{2y} =$</td>
</tr>
<tr>
<td>$a_{1y} =$</td>
<td></td>
</tr>
</tbody>
</table>

**Mathematical Analysis**
A mountaineer must leap across a 3.0 m wide crevasse. The other side of the crevasse is 4.0 m below the point from which the mountaineer leaps. The mountaineer leaps at 35° above horizontal and successfully makes the jump.

**Motion Diagram**

<table>
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</tr>
<tr>
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<td></td>
</tr>
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</table>

**Mathematical Analysis**
The right fielder flawlessly fields the baseball and throws a perfect strike to the catcher who tags out the base runner trying to score. The right fielder is approximately 300 feet (90 m) from home plate and throws the ball at an initial angle of 30° above horizontal. The catcher catches the ball on the fly exactly 1.7 m below the height from which it was thrown.

### Motion Diagram

#### Motion Information

<table>
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<tr>
<th>Event 1:</th>
<th>Event 2:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_1 =$</td>
<td>$t_2 =$</td>
</tr>
<tr>
<td>$r_{1x} =$</td>
<td>$r_{2x} =$</td>
</tr>
<tr>
<td>$r_{1y} =$</td>
<td>$r_{2y} =$</td>
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<tr>
<td>$v_{1x} =$</td>
<td>$v_{2x} =$</td>
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<tr>
<td>$v_{1y} =$</td>
<td>$v_{2y} =$</td>
</tr>
<tr>
<td>$a_{12x} =$</td>
<td>$a_{12y} =$</td>
</tr>
</tbody>
</table>

### Mathematical Analysis
A fire hose, with muzzle velocity of 24 m/s, is used to put out an apartment building fire. The fire is raging inside an apartment 5.0 m above the level of the hose and 10 m, measured horizontally, from the end of the hose. Ignore the effects of air resistance on the water.

**Motion Diagram**

**Motion Information**

<table>
<thead>
<tr>
<th>Event 1: Water leaves hose</th>
<th>Event 2: Water hits flames</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_1 = 0$ s</td>
<td>$t_2 = $</td>
</tr>
<tr>
<td>$r_{1x} = 0$ m</td>
<td>$r_{2x} = 10$ m</td>
</tr>
<tr>
<td>$r_{1y} = 0$ m</td>
<td>$r_{2y} = 5$ m</td>
</tr>
<tr>
<td>$v_{1x} = (24$ m/s$)\cos\theta$</td>
<td>$v_{2x} = $</td>
</tr>
<tr>
<td>$v_{1y} = (24$ m/s$)\sin\theta$</td>
<td>$v_{2y} = $</td>
</tr>
<tr>
<td>$a_{1x} = 0$ m/s$^2$</td>
<td>$a_{1y} = -9.8$ m/s$^2$</td>
</tr>
</tbody>
</table>

**Mathematical Analysis**

**x-direction**

\[
r_2 = r_1 + v_1(t_2 - t_1) + \frac{1}{2}a_{12}(t_2 - t_1)^2
\]

\[
10 = 0 + (24 \cos \theta) t_2 + 0
\]

\[
t_2 = \frac{10}{24 \cos \theta}
\]

**y-direction**

\[
r_2 = r_1 + v_1(t_2 - t_1) + \frac{1}{2}a_{12}(t_2 - t_1)^2
\]

\[
5 = 0 + (24 \sin \theta) t_2 + \frac{1}{2}(-9.8)t_2^2
\]

\[
5 = (24 \sin \theta)(\frac{10}{24 \cos \theta}) - 4.9(\frac{10}{24 \cos \theta})^2
\]

\[
5 = 10 \tan \theta - \frac{0.851}{\cos^2 \theta}
\]

\[
0 = 10 \tan \theta - \frac{0.851}{\cos^2 \theta} - 5
\]

This equation can be solved by using a “solver” program, by knowing a few trig identities, or, most conveniently, by graphing the righthand-side of the equation and finding where it crosses zero. The solution is $\theta = 32^\circ$.

**Therefore, $t_2 = 0.49$ s, and**

**x-direction**

\[
v_2 = v_1 + a_{12}(t_2 - t_1)
\]

\[
v_{2x} = 24 \cos 32^\circ + 0
\]

\[
v_{2x} = 20.4$ m/s$\]

**y-direction**

\[
v_2 = v_1 + a_{12}(t_2 - t_1)
\]

\[
v_{2y} = 24 \sin 32^\circ - 9.8(0.49)
\]

\[
v_{2y} = 7.92$ m/s$\]
A mountaineer must leap across a 3.0 m wide crevasse. The other side of the crevasse is 4.0 m below the point from which the mountaineer leaps. The mountaineer leaps at a speed of 3.5 m/s and barely makes the jump.

Motion Diagram

<table>
<thead>
<tr>
<th>Motion Information</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Event 1:</strong></td>
</tr>
<tr>
<td>$t_1 =$</td>
</tr>
<tr>
<td>$r_{1x} =$</td>
</tr>
<tr>
<td>$r_{1y} =$</td>
</tr>
<tr>
<td>$v_{1x} =$</td>
</tr>
<tr>
<td>$v_{1y} =$</td>
</tr>
<tr>
<td>$a_{1x} =$</td>
</tr>
<tr>
<td>$a_{1y} =$</td>
</tr>
</tbody>
</table>

| **Event 2:**        |
| $t_2 =$             |
| $r_{2x} =$          |
| $r_{2y} =$          |
| $v_{2x} =$          |
| $v_{2y} =$          |

Mathematical Analysis
The right fielder flawlessly fields the baseball and must throw a perfect strike to the catcher, 90 m away, to tag out the base runner trying to score. The right fielder knows she can throw a baseball at 80 mph (36 m/s) and calculates the proper angle at which to throw so that the catcher will catch the ball on the fly exactly 1.8 m below the height from which it was thrown. However, her calculation is so time-consuming that the ball arrives too late and the runner scores.

**Motion Diagram**

**Motion Information**

<table>
<thead>
<tr>
<th>Event 1</th>
<th>Event 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_1 =$</td>
<td>$t_2 =$</td>
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<tr>
<td>$r_{1x} =$</td>
<td>$r_{2x} =$</td>
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<td>$r_{1y} =$</td>
<td>$r_{2y} =$</td>
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<tr>
<td>$v_{1y} =$</td>
<td>$v_{2y} =$</td>
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</table>

**Mathematical Analysis**
At the buzzer, a basketball player shoots a desperation shot. She is 14 m from the basket and the ball leaves her hands exactly 1.4 m below the rim. She throws the ball at 18 m/s. Can she make the shot?

Motion Diagram

Motion Information

Event 1: Event 2:

\[ t_1 = \quad t_2 = \]
\[ r_{1x} = \quad r_{2x} = \]
\[ r_{1y} = \quad r_{2y} = \]
\[ v_{1x} = \quad v_{2x} = \]
\[ v_{1y} = \quad v_{2y} = \]
\[ a_{12x} = \]
\[ a_{12y} = \]

Mathematical Analysis
A ball is rolled off a level 0.80 m high table at 15 m/s. The floor beyond the table slopes down at a constant 5° below the horizontal.

### Motion Diagram

### Motion Information

<table>
<thead>
<tr>
<th>Event 1:</th>
<th>Event 2:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_1 = )</td>
<td>( t_2 = )</td>
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<tr>
<td>( r_{1x} = )</td>
<td>( r_{2x} = )</td>
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<tr>
<td>( r_{1y} = )</td>
<td>( r_{2y} = )</td>
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<tr>
<td>( v_{1x} = )</td>
<td>( v_{2x} = )</td>
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<tr>
<td>( v_{1y} = )</td>
<td>( v_{2y} = )</td>
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</tbody>
</table>

### Mathematical Analysis
A golf ball leaves the club at 18 m/s at an angle of 65° above the horizontal. The ground ahead slopes upward at 4°.

### Motion Diagram

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Event 1:</td>
</tr>
<tr>
<td>$t_1 =$</td>
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<tr>
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<tr>
<td>$r_{1y} =$</td>
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<tr>
<td>$v_{1x} =$</td>
</tr>
<tr>
<td>$v_{1y} =$</td>
</tr>
<tr>
<td>$a_{12x} =$</td>
</tr>
</tbody>
</table>

### Mathematical Analysis
In the shot put, the shot is accelerated from rest at 46 m/s\(^2\) over a distance of 0.90 m, oriented at 22° above horizontal, as the shot-putter extends his arm and launches the shot.

**Motion Information**

<table>
<thead>
<tr>
<th>Event 1: Begins to push</th>
<th>Event 2: Shot leaves shot.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_1 = 0 \text{ s} )</td>
<td>( t_2 = )</td>
</tr>
<tr>
<td>( r_{1x} = 0 \text{ m} )</td>
<td>( r_{2x} = (0.9 \text{ m}) \cos 22° )</td>
</tr>
<tr>
<td>( r_{1y} = 0 \text{ m} )</td>
<td>( r_{2y} = (0.9 \text{ m}) \sin 22° )</td>
</tr>
<tr>
<td>( v_{1x} = 0 \text{ m/s} )</td>
<td>( v_{2x} = )</td>
</tr>
<tr>
<td>( v_{1y} = 0 \text{ m/s} )</td>
<td>( v_{2y} = )</td>
</tr>
<tr>
<td>( a_{1x} = (46 \text{ m/s}^2) \cos 22° )</td>
<td>( v_{2x} = )</td>
</tr>
<tr>
<td>( a_{1y} = (46 \text{ m/s}^2) \sin 22° )</td>
<td></td>
</tr>
</tbody>
</table>

**x-direction**

\[
\begin{align*}
   r_2 &= r_1 + v_1 t_2 - t_1 + \frac{1}{2} a_{12} (t_2 - t_1)^2 \\
   0.834 &= 0 + 0 + \frac{1}{2} (42.7)t_2^2 \\
   t_2 &= 0.198 \text{s} \\

   v_2 &= v_1 + a_{12}(t_2 - t_1) \\
   v_{2x} &= 0 + 42.7(0.198) \\
   v_{2x} &= 8.44 \text{ m/s} \\

   v_2 &= v_1 + a_{12}(t_2 - t_1) \\
   v_{2y} &= 0 + 17.2(0.198) \\
   v_{2y} &= 3.41 \text{ m/s} 
\end{align*}
\]

Now that we’ve determined the speed of the shot as it leaves the shot putter’s hand, we could determine how far it goes if we knew the height of the shot when it left his hand.

**Motion Diagram**

We’ll soon learn a trick where we’ll rotate our coordinate system to simplify problems like this.

Don’t fall into the habit of always assuming \( a_x = 0 \text{ m/s}^2 \) and \( a_y = -9.8 \text{ m/s}^2 \). You are examining the motion of the shot while in the hand of the shot putter, not during freefall.
A kayaker 120 m east and 350 m north of home is moving with the current at 2 m/s to the south. He begins to paddle west, giving the kayak an acceleration of 0.2 m/s² for 15 s.

### Motion Diagram

### Motion Information

<table>
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<tr>
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<tr>
<td>( t_1 = )</td>
<td>( t_2 = )</td>
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<tr>
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<td>( r_{2x} = )</td>
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<tr>
<td>( r_{1y} = )</td>
<td>( r_{2y} = )</td>
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<td>( v_{1x} = )</td>
<td>( v_{2x} = )</td>
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<tr>
<td>( v_{1y} = )</td>
<td>( v_{2y} = )</td>
</tr>
<tr>
<td>( a_{12x} = )</td>
<td>( a_{12y} = )</td>
</tr>
</tbody>
</table>

### Mathematical Analysis
An astronaut on a spacewalk is 300 m from her spaceship and moving at 2.7 m/s, away from her ship at an angle of 17° from a line between her and the ship. She fires her jet pack for 4.0 s. The jetpack imparts an acceleration of 0.8 m/s² to her in the direction she was originally moving.

### Motion Diagram

<table>
<thead>
<tr>
<th>Event 1:</th>
<th>Event 2:</th>
</tr>
</thead>
<tbody>
<tr>
<td>t₁ =</td>
<td>t₂ =</td>
</tr>
<tr>
<td>r₁x =</td>
<td>r₂x =</td>
</tr>
<tr>
<td>r₁y =</td>
<td>r₂y =</td>
</tr>
<tr>
<td>v₁x =</td>
<td>v₂x =</td>
</tr>
<tr>
<td>v₁y =</td>
<td>v₂y =</td>
</tr>
</tbody>
</table>

### Mathematical Analysis

...
Determine the time-of-flight ($T$) of a rock thrown horizontally off of a cliff as a function of the initial velocity ($v_i$), the height of the cliff ($H$), and $g$. Assume the ground at the base of the cliff is level.

**Motion Information**

<table>
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<tr>
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</tr>
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<tr>
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<tr>
<td>$r_{1y} =$</td>
<td>$r_{2y} =$</td>
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<tr>
<td>$v_{1x} =$</td>
<td>$v_{2x} =$</td>
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<tr>
<td>$v_{1y} =$</td>
<td>$v_{2y} =$</td>
</tr>
<tr>
<td>$a_{12x} =$</td>
<td>$a_{12y} =$</td>
</tr>
</tbody>
</table>

**Mathematical Analysis**

**Questions**

If $H = \infty$, what should $T$ equal? Does your function agree with this observation?

If $g = 0 \text{ m/s}^2$, what should $T$ equal? Does your function agree with this observation?

If $v_i$ is doubled, what happens to $T$?
Determine the horizontal range (R) of a rock thrown horizontally off of a cliff as a function of the initial velocity (v_i), the height of the cliff (H), and g. Assume the ground at the base of the cliff is level.

**Motion Diagram**

<table>
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<tbody>
<tr>
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</tr>
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<td>v_{1y} =</td>
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<tr>
<td>a_{12x} =</td>
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<tr>
<td>a_{12y} =</td>
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<tr>
<td><strong>Event 2:</strong></td>
</tr>
<tr>
<td>t_2 =</td>
</tr>
<tr>
<td>r_{2x} =</td>
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<tr>
<td>r_{2y} =</td>
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<tr>
<td>v_{2x} =</td>
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<tr>
<td>v_{2y} =</td>
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</tbody>
</table>

**Mathematical Analysis**

**Questions**

*If H = ∞, what should R equal? Does your function agree with this observation?*

*If g = ∞, what should R equal? Does your function agree with this observation?*

*If v_i is doubled, what happens to R?*
Determine the maximum height (H) of a projectile launched over level ground as a function of the initial velocity (\(v_i\)), the launch angle (\(\theta\)), and g.

**Motion Diagram**

**Motion Information**

<table>
<thead>
<tr>
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<td>(r_{1y}) =</td>
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<td>(v_{1x}) =</td>
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<tr>
<td>(a_{1x}) =</td>
<td>(a_{1y}) =</td>
</tr>
</tbody>
</table>

**Mathematical Analysis**

**Questions**

*If g = \(\infty\), what should H equal? Does your function agree with this observation?*

*If \(\theta = 0^\circ\), what should H equal? Does your function agree with this observation?*

*If \(v_i\) is doubled, what happens to H?*
Determine the range (R) of a projectile launched over level ground as a function of the initial velocity (v_i), the launch angle (\( \theta \)), and g.

Motion Diagram

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</table>

Mathematical Analysis

Questions
If \( g = 0 \text{ m/s}^2 \), what should R equal? Does your function agree with this observation?

If \( \theta = 90^\circ \), what should R equal? Does your function agree with this observation?

If \( v_i \) is doubled, what happens to R?
A projectile is launched from the top of an decline of constant angle $\phi$. Determine the distance the projectile travels along the decline ($D$) as a function of the initial velocity ($v_i$), the launch angle above horizontal ($\theta$), the decline angle ($\phi$), and $g$.

**Motion Diagram**

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<tr>
<td>$a_{1y} =$</td>
<td>$a_{2y} =$</td>
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</tbody>
</table>

**Mathematical Analysis**

**Questions**

*If $\phi = 90^\circ$, what should $D$ equal? Does your function agree with this observation?*

*If $\theta = 90^\circ$, what should $D$ equal? Does your function agree with this observation?*
Dynamics

Concepts and Principles

Just like in kinematics, it’s an empirical fact about nature that when a force acts on an object in one direction (for example, the horizontal) this action does not appear to cause changes in the motion in a perpendicular direction (the vertical). Therefore, to investigate the effects of forces on the motion of an object in the vertical direction, you can ignore all forces acting in the horizontal direction. Of course, many forces will simultaneously act in both the horizontal and vertical directions. As in kinematics, the effect of these forces can be examined by concentrating on the components of the forces in the various directions. Again, as long as the directions of interest are perpendicular, the force components can be determined through right-angle trigonometry, and the magnitude of the force can always be determined by:

\[ F = \sqrt{F_x^2 + F_y^2 + F_z^2}. \]

Thus, Newton’s second law,

\[ \Sigma F = ma \]

is independently valid in any member of a set of perpendicular directions. The total force in the horizontal direction, for example, is equal to the mass times the acceleration in that direction. Note that the mass has been verified to be independent of direction, meaning that objects possess the same inertia in all directions.
**Dynamics**

*Analysis Tools*

**Drawing Free-Body Diagrams**

The free-body diagram is still the most important analysis tool for determining the forces that act on a particular object. As an example, start with a verbal description of a situation:

While rearranging furniture, a 600 N force is applied at an angle of 25° below horizontal to a 100 kg sofa at rest.

A free-body diagram for the sofa is sketched below:

![Free-Body Diagram of a Sofa]

The only non-contact interaction is the force of gravity, directed vertically downward.

The couch is in contact with two external objects, the person, pushing the couch across the floor, and the floor, exerting a force directed upward to prevent the couch from sinking into the floor. In addition, experience tells us that the interaction between the couch and the floor also hinders the motion of the couch in the direction of the person’s push. This portion of the couch-floor interaction is commonly referred to as friction.

This is a complete free-body diagram for the couch.

**The Force of Friction**

The interaction between objects in direct contact typically consists of two parts. One part of the interaction is directed perpendicular to the surface of contact. The other part of the interaction is the portion commonly called friction. The frictional portion of the interaction depends on many variables.

---

2 The portion of the interaction directed perpendicular to the surface of contact is sometimes referred to as the normal force, where normal has its mathematical definition of perpendicular.
For most situations, a model of friction limiting the number of variables effecting the interaction to two is adequate. These two variables are the magnitude of the perpendicular portion of the interaction, generically called the contact force, and a unit-less constant that reflects the relative roughness of the surface-to-surface contact, termed the coefficient of friction. This linear model of sliding friction further differentiates between the frictional interaction when the two surfaces are moving with respect to each other, termed kinetic friction, and when they are not, termed static friction.

**Kinetic friction**
The kinetic friction model states that the frictional interaction between the surfaces is approximately equal to the product of the contact force, $F_{contact}$ and the coefficient of friction for kinetic situations, $\mu_k$;

$$F_{friction} = \mu_k F_{contact}$$

The direction of this force on a particular object is in opposition to the relative motion of the two surfaces in contact.

**Static friction**
The static friction model states that the frictional interaction between the surfaces must be less than, or at most equal to, the product of the contact force, $F_{contact}$ and the coefficient of friction for static situations, $\mu_s$;

$$F_{friction} \leq \mu_s F_{contact}$$

The direction of this force on a particular object is in opposition to the motion that would result if the frictional interaction were not present.

---

**Applying Newton’s Second Law**

Using this model for friction, we can now quantitatively analyze the original situation. Note that the two coefficients of friction will typically be given as an ordered pair, $(\mu_s, \mu_k)$.

*While rearranging furniture, a 600 N force is applied at an angle of 25° below horizontal to a 100 kg sofa at rest. The coefficient of friction between the sofa and the floor is (0.5, 0.4).*
A free-body diagram for the sofa is sketched below:

![Free-body diagram of a sofa](image)

Hopefully you realize that two quite different outcomes can result from this push. Either the person pushes too weakly to move the couch or the push is sufficient to make the couch move. Since the frictional forces acting in these two cases are quite different, we can’t really numerically analyze the situation until we make an assumption as to the outcome of the push. Of course, we will then have to check the validity of our assumption once we have completed our analysis. If our assumption turns out to be incorrect, we will then have to re-analyze the situation using the other possible outcome.

I will assume the couch doesn’t move for the analysis below, and then later check the assumption. Assuming the couch doesn’t move is equivalent to assuming $a_x = 0 \text{ m/s}^2$ and that the relevant type of friction to use is static friction.

Applying Newton’s Second Law independently in the horizontal (x) and vertical (y) directions yields:

**x-direction**

$$\Sigma F = ma$$

$$F_{\text{person}} \cos 25 - F_{\text{static friction}} = ma_x$$

$$600 \cos 25 - F_{\text{static friction}} = 100(0)$$

$$F_{\text{static friction}} = 544 \text{ N}$$

**y-direction**

$$\Sigma F = ma$$

$$- F_{\text{person}} \sin 25 - F_{\text{gravity}} + F_{\text{floor}} = ma_y$$

$$- 600 \sin 25 - (100)(9.8) + F_{\text{floor}} = 100(0)$$

$$F_{\text{floor}} = 1234 \text{ N}$$

Notice that the acceleration of the couch in the vertical direction must be zero regardless of my assumption, unless the couch begins to levitate or crash through the floor.

Assuming the couch doesn’t move leads to a calculated value of static friction equal to $544 \text{ N}$. Can static friction create a force of this magnitude to prevent the couch’s motion? I can check this calculated value against the allowed values for static friction:

$$F_{\text{static friction}} \leq \mu_s F_{\text{contact}}$$

$$F_{\text{static friction}} \leq (0.5)(1234)$$

$$F_{\text{static friction}} \leq 617 \text{ N}$$
Since the calculated value of the static frictional force is below the maximum possible value of the static frictional force, my analysis and assumption are valid, the couch does not budge. The person is not pushing hard enough to overcome the static frictional force that acts to prevent the couch’s motion relative to the floor.

Therefore, in this scenario the actual value of the static frictional force is 544 N (remember, it can be any value less than or equal to 617 N) and the acceleration of the couch is equal to zero.

How would the analysis change if the couch was initially in motion? Assume you enlisted a friend to help get the couch moving, but as soon as it began to move your friend stopped pushing. Would the couch stop immediately, gradually slow down to a stop, or could you keep the couch in motion across the room?

If the couch was initially moving, two things must change in our analysis. First, the horizontal acceleration of the couch is no longer necessarily zero. Second, the frictional force acting on the couch is kinetic.

Applying Newton’s Second Law independently in the horizontal (x) and vertical (y) directions now yields:

<table>
<thead>
<tr>
<th>x-direction</th>
<th>y-direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \Sigma F = ma ]</td>
<td>[ \Sigma F = ma ]</td>
</tr>
<tr>
<td>[ F_{\text{person}} \cos 25 - F_{\text{kin friction}} = 100a_x ]</td>
<td>[ F_{\text{person}} \sin 25 - F_{\text{gravity}} + F_{\text{floor}} = ma_y ]</td>
</tr>
<tr>
<td>[ 600 \cos 25 - F_{\text{kin friction}} = 100a_x ]</td>
<td>[ -600 \cos 25 - (100)(9.8) + F_{\text{floor}} = 100(0) ]</td>
</tr>
<tr>
<td>[ 544 - F_{\text{kin friction}} = 100a_x ]</td>
<td>[ F_{\text{floor}} = 1234 \text{ N} ]</td>
</tr>
</tbody>
</table>

To finish the analysis, we need to calculate the kinetic frictional force.

\[ F_{\text{kin friction}} = \mu_k F_{\text{contact}} \]

\[ F_{\text{kin friction}} = (0.4)(1234) \]

\[ F_{\text{kin friction}} = 494 \text{ N} \]

Substituting this into the x-equation above yields:

\[ 544 - 494 = 100a_x \]
\[ 50 = 100a_x, \quad 50 = 100a_x \]
\[ a_x = 0.50 \text{ m/s}^2 \]

Thus, if the couch is already moving the kinetic frictional force is 494 N and the couch accelerates toward the right at 0.50 m/s². In summary, if the couch is initially moving it will continue to move and accelerate at 0.50 m/s² to the right. If it is initially at rest, the person pushing on it will not be able to get it to move.
Choosing a Coordinate System

In analyzing a scenario, you are always free to choose whatever coordinate system you like. If you make up negative, or left positive, this will not make you get the wrong answer. However, certain coordinate systems may make the mathematical analysis simpler than other coordinate systems. For example;

A 75 kg skier starts from rest at the top of a 20° slope. He's a show-off, so he skies down the hill backward. The frictional coefficient between his skies and the snow is (0.10,0.05).

In attempting to analyze this situation, first draw a free-body diagram.

Notice that I have chosen the traditional horizontal and vertical coordinate system. I could analyze the situation using this coordinate system, but there are two difficulties with this choice.

1. Neither the force of the surface nor the force of friction is oriented in the x- or y-direction. (The force of gravity is oriented in the negative y-direction.) Therefore, I will have to use trigonometry to determine the x- and y-components of both of these forces.

2. The skier is accelerating down the inclined slope. Thus, I will also need trigonometry to determine the x- and y-components of the acceleration.

Although these difficulties are by no means insurmountable, why make the task more difficult than it has to be?

Contrast the above choice of coordinate system with a coordinate system in which the x-direction is tilted parallel to the surface on which the skier slides and the y-direction, remaining perpendicular to the x, is perpendicular to the surface.

1. Using the tilted coordinate system, the only force not oriented in the x- or y-direction is the force of gravity. Therefore, I will only need to use trigonometry to determine the x- and y-components of one force rather than two.

2. The skier is accelerating down the inclined slope. Since the x-direction is oriented parallel to the slope, the skier has an acceleration in the x-direction and zero acceleration in the y-direction.
This simple rotation of the coordinate system has made the mathematical analysis of this situation much easier. Applying Newton’s second law in the x- and y-direction leads to:

**x-direction**

\[ \Sigma F = m a \]

\[ F_{\text{gravity}} (\sin 20) - F_{\text{friction}} = m a_x \]

\[ (75)(9.8)(\sin 20) - F_{\text{friction}} = 100 a_x \]

\[ 251 - F_{\text{friction}} = 100 a_x \]

**y-direction**

\[ \Sigma F = m a \]

\[ -F_{\text{gravity}} (\cos 20) + F_{\text{surface}} = m (0) \]

\[ -(75)(9.8)(\cos 20) + F_{\text{surface}} = m (0) \]

\[ F_{\text{surface}} = 691 \text{ N} \]

Notice that if the x-axis is rotated by 20° from horizontal to become parallel to the slope, the y-axis is rotated by 20° from vertical. Since the force of gravity is always oriented vertically downward, it's now 20° from the y-axis.

Thus, the force of gravity has a component in the positive x-direction of \( F_{\text{gravity}} (\sin 20°) \) and a component in the negative y-direction of \( F_{\text{gravity}} (\cos 20°) \).

Now that the contact force between the skier and the slope is known, the static friction force can be determined.

\[ F_{\text{friction}} \leq \mu_s F_{\text{contact}} \]

\[ F_{\text{friction}} \leq (0.10)(691) \]

\[ F_{\text{friction}} \leq 69 \text{ N} \]

Since the x-component of the force of gravity on the skier (251 N) is larger than the force of static friction (69 N), the skier will accelerate down the hill. Once he begins to move, the frictional force must be calculated using the kinetic friction model.

\[ F_{\text{friction}} = \mu_k F_{\text{contact}} \]

\[ F_{\text{friction}} = (0.05)(691) \]

\[ F_{\text{friction}} = 35 \text{ N} \]

Examining the x-component of Newton’s second law:
$251 - F_{\text{friction}} = 100\ a_x$

$251 - 35 = 100\ a_x$

$a_x = 2.2\ \text{m/s}^2$

The skier accelerates down the slope with an acceleration of 2.2 m/s$^2$. 
Dynamics

Activities
Construct free-body diagrams for the objects described below.

a. Someone mistakenly put a lovely couch out at the curb on garbage day and you decide to take it back to your apartment. You push horizontally on the 80 kg couch with a force of 320 N. The frictional coefficient is (0.40,0.35).

   assuming the couch does not move
   assuming the couch does move

   ![Couch Diagrams](image)

b. A 100 kg bicycle and rider initially move at 16 m/s up a 15° hill. The rider slams on the brakes and skids to rest. The coefficient of friction is (0.8,0.7).

   while skidding
   when the bike is at rest on the incline

   ![Bicycle Diagrams](image)

c. A 10 kg box is stacked on top of a 25 kg box. The boxes are at rest on an 8° incline.

   the top box
   the bottom box

   ![Box Diagrams](image)
Construct free-body diagrams for the objects described below.

a. Someone mistakenly put a lovely couch out at the curb on garbage day and you decide to take it back to your apartment. You pull on the 110 kg couch with a force of 410 N directed at 35° above horizontal. The frictional coefficient is (0.40, 0.35).

assuming the couch does not move

assuming the couch does move

b. A 60 kg skier starts from rest at the top of a 100 m, 25° slope. He doesn’t push with his poles because he’s afraid of going too fast. The frictional coefficient is (0.10, 0.05).

while skiing downhill

while being pulled back uphill by the towrope

c. A 10 kg box is stacked on top of a 25 kg box. The boxes are sliding down an 18° incline at increasing speed. The top box is not moving relative to the bottom box.

the top box

the bottom box
The strange man below is trying to pull the pair of boxes up the incline. Construct the requested free-body diagrams.

a. The boxes *almost* move up the incline.

\[
\begin{align*}
\text{the top box} & \\
\text{the bottom box}
\end{align*}
\]

b. The boxes move up the incline.

\[
\begin{align*}
\text{the top box} & \\
\text{the bottom box}
\end{align*}
\]

c. The bottom box moves up the incline but the top box slides off the bottom box.

\[
\begin{align*}
\text{the top box} & \\
\text{the bottom box}
\end{align*}
\]
The strange man below is trying to prevent himself from getting crushed by the boxes. Construct the requested free-body diagrams.

a. The boxes *almost* move down the incline.

```
the top box

the bottom box
```

b. The boxes *almost* move up the incline.

```
the top box

the bottom box
```

c. The bottom box *almost* moves down the incline but the top box slides off the bottom box.

```
the top box

the bottom box
```
A constant magnitude force is applied to a rope attached to a crate. The crate is on a level surface. For each of the following situations, circle the correct relationship symbol between the two force magnitudes and explain your reasoning.

a. The crate moves at constant speed and the rope is horizontal.

\[
\begin{align*}
F_{\text{gravity}} & > = < \ F_{\text{surface}} \\
F_{\text{rope}} & > = < \ F_{\text{friction}}
\end{align*}
\]

Explanation:

b. The crate does not move and the rope is horizontal.

\[
\begin{align*}
F_{\text{gravity}} & > = < \ F_{\text{surface}} \\
F_{\text{rope}} & > = < \ F_{\text{friction}}
\end{align*}
\]

Explanation:

c. The crate moves at constant speed and the rope is inclined above the horizontal.

\[
\begin{align*}
F_{\text{gravity}} & > = < \ F_{\text{surface}} \\
F_{\text{rope}} & > = < \ F_{\text{friction}}
\end{align*}
\]

Explanation:
A constant magnitude force is applied to a rope attached to a crate. The crate is on an inclined surface. For each of the following situations, circle the correct relationship symbol between the two force magnitudes and explain your reasoning.

a. The crate does not move and the rope is parallel to the incline and directed up the incline.

\[ F_{\text{gravity}} \quad > \quad = \quad < \quad ? \quad F_{\text{surface}} \]
\[ F_{\text{rope}} \quad > \quad = \quad < \quad ? \quad F_{\text{friction}} \]

Explanation:

b. The crate does not move and the rope is parallel to the incline and directed down the incline.

\[ F_{\text{gravity}} \quad > \quad = \quad < \quad ? \quad F_{\text{surface}} \]
\[ F_{\text{rope}} \quad > \quad = \quad < \quad ? \quad F_{\text{friction}} \]

Explanation:

c. The crate moves at constant speed up the incline and the rope is parallel to the incline and directed up the incline.

\[ F_{\text{gravity}} \quad > \quad = \quad < \quad ? \quad F_{\text{surface}} \]
\[ F_{\text{rope}} \quad > \quad = \quad < \quad ? \quad F_{\text{friction}} \]

Explanation:
Below are six crates at rest on level surfaces. The crates have different masses and the frictional coefficients between the crates and the surfaces differ. The same external force is applied to each crate, but none of the crates move. Rank the crates on the basis of the magnitude of the frictional force acting on them.

Largest  1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest  _____

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
Below are six crates at rest on level surfaces. The masses, frictional coefficients between the crates and the surfaces, and the external applied force all differ.

a. If none of the crates move, rank the crates on the basis of the magnitude of the frictional force acting on them.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

b. If the crates are moving, rank the crates on the basis of the magnitude of the frictional force acting on them.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
Below are six boxes held at rest against a wall. The coefficients of friction between each box and the wall are identical.

a. Rank the boxes on the basis of the magnitude of the force of the wall acting on them.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest _____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

b. Rank the boxes on the basis of the magnitude of the frictional force acting on them.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest _____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
Below are eight crates of differing mass. The frictional coefficients between each crate and the surface on which they slide are the same. Each crate is being pulled to the right at constant speed. All crates travel at the same constant speed. Rank the magnitude of the force exerted by each rope on the crate immediately to its left.

Largest 1.  2.  3.  4.  5.  6.  7.  8.  Smallest

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
Below are eight crates of differing mass. The frictional coefficients between each crate and the surface on which they slide are so small that the force of friction is negligible on all crates. Each crate is being pulled to the right and accelerating. The acceleration of each crate or chain of crates is given. Rank the magnitude of the force exerted by each rope on the crate immediately to its left.

Largest
_____

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
Someone mistakenly put a lovely couch out at the curb on garbage day and you decide to take it back to your apartment. You push on the 110 kg couch with a force of 410 N directed at 35° below horizontal. The couch doesn’t move.

Free-Body Diagram

Mathematical Analysis
Someone mistakenly put a lovely couch out at the curb on garbage day and you decide to take it back to your apartment. You push on the 80 kg couch with a force of 320 N directed at 15° below horizontal. The frictional coefficient is (0.40, 0.35).

Free-Body Diagram

Mathematical Analysis
Someone mistakenly put a lovely couch out at the curb on garbage day and you decide to take it back to your apartment. You pull on the 110 kg couch with a force of 510 N directed at 35° above horizontal. The frictional coefficient is (0.40, 0.35).

Free-Body Diagram

Mathematical Analysis
You get into a fight with another person over a garbage-day couch. You push on the 80 kg couch with a force of 660 N directed at 15° below horizontal. She claims ownership by sitting on the couch while you try to push it. You still manage to just barely get the couch moving. The frictional coefficient is (0.40,0.35).
You get into a fight with another person over a garbage-day couch. You push on the 80 kg couch with a force of 420 N directed at 15° below horizontal. She pushes on the other side of the couch with a force of 510 N directed at 25° below horizontal. The frictional coefficient is (0.40, 0.35).
The person at right exerts the minimum force necessary to support the 100 kg block. He pushes at an angle of 50° above the horizontal. The coefficient of friction is (0.6,0.5).

Since we are looking for the minimum force needed to hold the block in place, the block is almost moving downward. This means that the frictional force is static and directed upward.

Free-Body Diagram

Mathematical Analysis

**x-direction**

\[ F_{\text{push}} \cos 50 - F_{\text{wall}} = 100(0) \]

\[ F_{\text{wall}} = 0.643 F_{\text{push}} \]

**y-direction**

\[ F_{\text{push}} \sin 50 - (100)(9.8) + F_{\text{static friction}} = 100(0) \]

\[ F_{\text{static friction}} \leq 0.6 F_{\text{wall}} \]

\[ F_{\text{static friction}} \leq 0.6(0.643 F_{\text{push}}) \]

\[ F_{\text{static friction}} \leq 0.386 F_{\text{push}} \]

Since we are looking for the minimum force needed to support the block, the block is almost moving downward. This means that static friction is at its maximum value. Therefore we can substitute 0.386\( F_{\text{push}} \) into the y-equation and solve.

\[ 0.766 F_{\text{push}} - 980 + 0.386 F_{\text{push}} = 0 \]

\[ 1.152 F_{\text{push}} = 980 \]

\[ F_{\text{push}, \min} = 851 N \]

If we were looking for the maximum force, everything would be the same except for the direction of the frictional force (it would be downward). Therefore, the maximum force would be

\[ 0.766 F_{\text{push}} - 980 - 0.386 F_{\text{push}} = 0 \]

\[ 0.380 F_{\text{push}} = 980 \]

\[ F_{\text{push}, \max} = 2579 N \]
The person at right exerts a 850 N force on the 90 kg block at an angle of 55° above the horizontal. The coefficient of friction is (0.6, 0.5).
The person at right exerts a 620 N force on the 70 kg block at an angle of 40° above the horizontal. The coefficient of friction is (0.5, 0.4).
A boy pulls a 30 kg sled, including the mass of his kid brother, along ice. The boy pulls on the tow rope, oriented at 60° above horizontal, with a force of 110 N until his kid brother begins to cry. Like clockwork, his brother always cries upon reaching a speed of 2.0 m/s. The frictional coefficient is (0.20,0.15).

**Motion Information**

**Free-Body Diagram**

<table>
<thead>
<tr>
<th>Event 1:</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$t_1 =$</td>
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<td>$r_{2x} =$</td>
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<td>$r_{1y} =$</td>
<td>$r_{2y} =$</td>
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<td>$a_{12x} =$</td>
<td>$v_{2y} =$</td>
</tr>
<tr>
<td>$a_{12y} =$</td>
<td></td>
</tr>
</tbody>
</table>

**Mathematical Analysis**
Starting from rest, a girl can pull a sled, carrying her kid brother, 20 m in 8 s. The girl pulls on the tow rope, oriented at 30° above horizontal, with a force of 90 N. The frictional coefficient is (0.15, 0.10).

### Motion Information

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<td>$a_{2y} =$</td>
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</table>

### Free-Body Diagram

![Free-Body Diagram](image_url)

### Mathematical Analysis
A 60 kg skier starts from rest at the top of a 100 m, 25° slope. He doesn’t push with his poles because he’s afraid of going too fast. The frictional coefficient is (0.10, 0.05).

Motion Information

Event 1:
- \( t_1 = \)
- \( r_{1x} = \)
- \( r_{1y} = \)
- \( v_{1x} = \)
- \( v_{1y} = \)
- \( a_{1x} = \)
- \( a_{1y} = \)

Event 2:
- \( t_2 = \)
- \( r_{2x} = \)
- \( r_{2y} = \)
- \( v_{2x} = \)
- \( v_{2y} = \)

Mathematical Analysis
A 100 kg bicycle and rider initially move at 16 m/s up a 15° hill. The rider slams on the brakes and skids to rest. The coefficient of friction is (0.8, 0.7).

Motion Information

Event 1:  
- \( t_1 = \)  
- \( r_{1x} = \)  
- \( r_{1y} = \)  
- \( v_{1x} = \)  
- \( v_{1y} = \)  
- \( a_{1x} = \)  
- \( a_{1y} = \)

Event 2:  
- \( t_2 = \)  
- \( r_{2x} = \)  
- \( r_{2y} = \)  
- \( v_{2x} = \)  
- \( v_{2y} = \)  
- \( a_{2x} = \)  
- \( a_{2y} = \)

Free-Body Diagram

Mathematical Analysis
A 70 kg snowboarder starts from rest at the top of a 270 m, 20° slope. She reaches the bottom of the slope in 14.5 seconds.

**Motion Information**

Event 1:

- \( t_1 = \) 
- \( r_{1x} = \) 
- \( r_{1y} = \) 
- \( v_{1x} = \) 
- \( v_{1y} = \) 
- \( a_{1x} = \) 
- \( a_{1y} = \)

Event 2:

- \( t_2 = \) 
- \( r_{2x} = \) 
- \( r_{2y} = \) 
- \( v_{2x} = \) 
- \( v_{2y} = \) 
- \( a_{2x} = \) 
- \( a_{2y} = \)

**Free-Body Diagram**

---

**Mathematical Analysis**
At a UPS distribution center, a 60 kg crate is at rest on an 8° ramp. A worker applies the minimum horizontal force needed to push the crate up the ramp. The coefficient of friction between the crate and the ramp is \((0.3, 0.2)\).
At a UPS distribution center, a 40 kg crate is sliding down an 8° ramp at 3 m/s. A worker applies a horizontal force to the crate and brings the crate to rest in 1.5 s. The coefficient of friction between the crate and the ramp is (0.3, 0.2).

**Motion Information**

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<tr>
<td>$v_{1y}$</td>
<td>$v_{2y}$</td>
</tr>
</tbody>
</table>

**Free-Body Diagram**

**Mathematical Analysis**
The device at right guarantees all the excitement of skiing without the need for hills. The skier begins from rest 35 m from the brick wall. The block has a mass of 50 kg and the skier has a mass of 75 kg. The coefficient of friction is (0.15, 0.13).

I'm choosing a coordinate system where the +x-direction is the direction of motion of the system. This means that “right” is +x for the skier and “down” is +x for the block!

**Motion Information**

Object: Skier

Event 1: Block is released

Event 2: Smashes into wall

<table>
<thead>
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<tr>
<td>( t_1 = 0 \text{ s} )</td>
<td>( t_2 = )</td>
</tr>
<tr>
<td>( r_{1x} = 0 \text{ m} )</td>
<td>( r_{2x} = 35 \text{ m} )</td>
</tr>
<tr>
<td>( r_{1y} = 0 \text{ m} )</td>
<td>( r_{2y} = 0 \text{ m} )</td>
</tr>
<tr>
<td>( v_{1x} = 0 \text{ m/s} )</td>
<td>( v_{2x} = )</td>
</tr>
<tr>
<td>( v_{1y} = 0 \text{ m/s} )</td>
<td>( v_{2y} = 0 \text{ m} )</td>
</tr>
<tr>
<td>( a_{1x} = )</td>
<td>( a_{1y} = 0 \text{ m/s}^2 )</td>
</tr>
</tbody>
</table>

**Mathematical Analysis**

**Skier**

**y-direction**

\[
F_{\text{surface}} = -75(9.8) = 75(0)
\]

\[
F_{\text{surface}} = 735N
\]

**friction**

\[
F_{\text{kinetic friction}} = 0.13F_{\text{surface}}
\]

\[
F_{\text{kinetic friction}} = 0.13(735)
\]

\[
F_{\text{kinetic friction}} = 95.6N
\]

**x-direction**

\[
F_{\text{friction}} = 75a_{\text{skier}}
\]

\[
F_{\text{rope}} - F_{\text{friction}} = 75a_{\text{skier}}
\]

\[
F_{\text{rope}} - 95.6 = 75a_{\text{skier}}
\]

**Block**

**x-direction**

\[
-F_{\text{rope}} + 50(9.8) = 50a_{\text{block}}
\]

\[
-F_{\text{rope}} + 490 = 50a_{\text{block}}
\]

In the coordinate system above, \( a_{\text{skier}} = a_{\text{block}} = a \), so the two \( x \)-equations can be added to yield:

\[
F_{\text{rope}} - 95.6 = 75a_{\text{skier}}
\]

\[
394.4 = 125a
\]

\[
a = 3.16m/s^2
\]

Kinematics can be used to find \( v_x = 14.9 \text{ m/s} \) and \( t_2 = 4.71 \text{ s} \).
The device at right allows novices to ski downhill at reduced speeds. The block has a mass of 15 kg and the skier has a mass of 70 kg. The coefficient of friction is (0.05, 0.04). The skier starts from rest at the top of a 30 m, 20° slope.

### Motion Information

**Object:**

**Event 1:**

<table>
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<table>
<thead>
<tr>
<th>$r_{1x} =$</th>
<th>$r_{2x} =$</th>
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<th>$a_{12x} =$</th>
<th>$v_{2y} =$</th>
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<th>$a_{12y} =$</th>
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### Free-Body Diagrams

**Skier**

**Block**

### Mathematical Analysis
The device at right allows you to ski uphill. The ballast block has a mass of 30 kg and the skier has a mass of 60 kg. The coefficient of friction is (0.07, 0.06). The skier starts from rest at the bottom of a 30 m, 20° slope.

**Motion Information**

Object:

Event 1:

- \( t_1 = \) 
- \( r_{1x} = \) 
- \( r_{1y} = \) 
- \( v_{1x} = \) 
- \( v_{1y} = \) 
- \( a_{1x} = \) 
- \( a_{1y} = \)

Event 2:

- \( t_2 = \) 
- \( r_{2x} = \) 
- \( r_{2y} = \) 
- \( v_{2x} = \) 
- \( v_{2y} = \)

**Free-Body Diagrams**

- skier
- block

**Mathematical Analysis**
The device at right may allow you to ski uphill (or it may allow you to ski downhill backward). The ballast block has a mass of 20 kg and the skier has a mass of 70 kg. The coefficient of friction is (0.1, 0.09). The ramp is inclined at 20° above horizontal.

Free-Body Diagrams

Mathematical Analysis
The strange man at right wants to pull the two blocks to the other side of the room in as short a time as possible. However, he doesn’t want the top block to slide relative to the bottom block. The coefficient of friction between the bottom block and the floor is (0.25,0.20) and the coefficient of friction between the top block and the bottom block is (0.30,0.25). The blocks start from rest.

Free-Body Diagrams

top block

bottom block

Mathematical Analysis
The strange man at right wants to pull the two blocks to the other side of the room in as short a time as possible by pulling on the top block. However, he doesn't want the top block to slide relative to the bottom block. The coefficient of friction between the bottom block and the floor is \(0.10, 0.05\) and the coefficient of friction between the top block and the bottom block is \(0.60, 0.50\). The blocks start from rest.

Free-Body Diagrams

\[
\begin{align*}
\text{top block} \\
\text{bottom block}
\end{align*}
\]
The strange man at right wants to pull the two blocks to the top of the hill in as short a time as possible. However, he doesn’t want the top block to slide relative to the bottom block. The coefficient of friction between the 150 kg bottom block and the floor is (0.25,0.20) and the coefficient of friction between the 50 kg top block and the bottom block is (0.30,0.25). The hill is inclined at 15° above horizontal. The blocks start from rest.

**Free-Body Diagrams**

*top block*  
*bottom block*

**Mathematical Analysis**
The strange man at right applies the minimum force necessary to not get crushed by the bottom block. (The top block may or may not crush him.) The coefficient of friction between the 150 kg bottom block and the floor is (0.25, 0.20) and the coefficient of friction between the 50 kg top block and the bottom block is (0.40, 0.35). The hill is inclined at 20° above horizontal. The blocks are initially at rest.

Free-Body Diagrams

Mathematical Analysis
You should know the story by now. You push on a garbage-day couch at an angle $\theta$ below horizontal. Determine the minimum force ($F_{\text{min}}$) needed to move the couch as a function of the couch’s mass ($m$), $\theta$, the appropriate coefficient of friction, and $g$.

**Questions**

*If $\theta = 0^\circ$, what should $F_{\text{min}}$ equal? Does your function agree with this observation?*

*If $\theta = 90^\circ$, what should $F_{\text{min}}$ equal? Does your function agree with this observation?*

*If $m = \infty$, what should $F_{\text{min}}$ equal? Does your function agree with this observation?*
The man at right exerts a force on the block at an angle $\theta$ above horizontal. Determine the minimum force ($F_{\text{min}}$) needed to begin to slide the block up the wall as a function of the block’s mass ($m$), $\theta$, the appropriate coefficient of friction, and $g$.

**Questions**

*If $\theta = 90^\circ$, what should $F_{\text{min}}$ equal? Does your function agree with this observation?*

*If $\theta = 0^\circ$, what should $F_{\text{min}}$ equal? Does your function agree with this observation?*

*Below what angle $\theta$ is it impossible to slide the block up the wall?*
A crate is held at rest on a ramp inclined at \( \theta \) from horizontal. Determine the minimum force \( F_{\text{min}} \), applied parallel to the incline, needed to prevent the crate from sliding down the ramp as a function of the crate’s mass \( m \), \( \theta \), the appropriate coefficient of friction, and \( g \).

**Free-Body Diagram**

**Mathematical Analysis**

**Questions**

If \( \theta = 0^\circ \), what should \( F_{\text{min}} \) equal? Does your function agree with this observation?

If \( \theta = 90^\circ \), what should \( F_{\text{min}} \) equal? Does your function agree with this observation?

If \( m = \infty \), what should \( F_{\text{min}} \) equal? Does your function agree with this observation?
A crate is held at rest on a ramp inclined at \( \theta \) from horizontal. Determine the maximum force \( (F_{\text{max}}) \), applied horizontally, before the crate begins to move as a function of the crate’s mass \( (m) \), \( \theta \), the appropriate coefficient of friction, and \( g \).

**Questions**

*If \( m = \infty \), what should \( F_{\text{max}} \) equal? Does your function agree with this observation?*

*If \( g = \infty \), what should \( F_{\text{min}} \) equal? Does your function agree with this observation?*

*If \( \theta = 0^\circ \), what should \( F_{\text{max}} \) equal? Does your function agree with this observation?*
A skier of mass $m$ starts from rest at the top of a ski run of incline $\theta$. Determine the minimum angle ($\theta_{\text{min}}$) such that the skier will begin to slide down the slope without pushing off as a function of $m$, the appropriate coefficient of friction, and $g$.

**Questions**

*If $\mu = 0$, what should $\theta_{\text{min}}$ equal? Does your function agree with this observation?*

*If $g = 0 \text{ m/s}^2$, what should $\theta_{\text{min}}$ equal? Does your function agree with this observation?*

*If $m$ was twice as large, what should $\theta_{\text{min}}$ equal? Does your function agree with this observation?*
Conservation Laws

Concepts and Principles

The Impulse-Momentum Relation

Just like the kinematic relations and Newton’s second law, the impulse-momentum relation is independently valid in any member of a set of perpendicular directions. Thus, we will typically apply the impulse-momentum relation in its component forms:

\[
\begin{align*}
mv_{xi} + \Sigma(F_x(\Delta t)) &= mv_{xf} \\
mv_{yi} + \Sigma(F_y(\Delta t)) &= mv_{yf} \\
mv_{zi} + \Sigma(F_z(\Delta t)) &= mv_{zf}
\end{align*}
\]

The Work-Energy Relation

From Model 1, our expression for the Work-Energy Relation, with gravitational potential energy terms, is:

\[
\frac{1}{2}mv_i^2 + mgh_i + \Sigma(F_{||}r|\cos\phi) = \frac{1}{2}mv_f^2 + mgh_f
\]

It’s very important to remember that the work-energy relation is a scalar equation, meaning it cannot be broken into components and “solved” separately in the x-, y-, and z-directions. This is even more important to remember now that we are working in multiple dimensions. This observation results in two important points:

- The work-energy relation involves the actual initial and final velocities, not their components. The kinetic energy of an object does not depend on the direction of travel of the object.

- In the expression for work, \(F_{||}r|\cos\phi\), the product of the magnitude of the force and the magnitude of the displacement is multiplied by \(\cos\phi\), where \(\phi\) is defined to be the angle between the applied force and the displacement of the object. If the force and displacement are in the same direction \(\phi = 0^\circ\), and the work is positive (the object gains energy). If the force and displacement are in the opposite direction \(\phi = 180^\circ\), and the work is negative (the object loses energy). If the force and displacement are perpendicular, no work is done. Note that the actual directions of the force and the displacement are unimportant, only their directions relative to each other affect the work.
**Conservation Laws**

**Analysis Tools**

### Applying the Impulse-Momentum Relation to a Single Object

Let’s investigate the following scenario:

A boy pulls a 30 kg sled, including the mass of his kid brother, along ice. The boy pulls on the tow rope, oriented at 60° above horizontal, with a force of 110 N until his kid brother begins to cry. Like clockwork, his brother always cries upon reaching a speed of 2.0 m/s. The frictional coefficient is (0.20, 0.15).

Let’s apply the impulse-momentum relations to see what they reveal about the situation.

Applying impulse-momentum separately in the x- and y-directions yields:

#### x-direction

\[ mv_{x_i} + \sum (F_x(\Delta t)) = mv_{x_f} \]

\[
30(0) + F_{\text{rope}} \cos 60(\Delta t) - F_{\text{kinetic friction}}(\Delta t) = 30(2)
\]

\[
0 + 110 \cos 60(\Delta t) - 0.15 F_{\text{surface}}(\Delta t) = 60
\]

\[
55(\Delta t) - 0.15 F_{\text{surface}}(\Delta t) = 60
\]

#### y-direction

\[ mv_{y_i} + \sum (F_y(\Delta t)) = mv_{y_f} \]

\[
30(0) + F_{\text{rope}} \sin 60(\Delta t) - F_{\text{gravity}}(\Delta t) + F_{\text{surface}}(\Delta t) = 30(0)
\]

\[
0 + 110 \sin 60(\Delta t) - (30)(9.8)(\Delta t) + F_{\text{surface}}(\Delta t) = 0
\]

\[
95(\Delta t) - 294(\Delta t) + F_{\text{surface}}(\Delta t) = 0
\]

\[
95 - 294 + F_{\text{surface}} = 0
\]

\[
F_{\text{surface}} = 199 N
\]
Substituting the value for the force of the surface into the x-equation,

\[ 55(\Delta t) - 0.15(199)(\Delta t) = 60 \]
\[ 55(\Delta t) - 30(\Delta t) = 60 \]
\[ 25(\Delta t) = 60 \]
\[ \Delta t = 2.4 \text{s} \]

The kid brother begins to cry after only 2.4 s.

---

### Applying the Work-Energy Relation to a Single Object

What will the work-energy relation tell us about the same scenario?

A boy pulls a 30 kg sled, including the mass of his kid brother, along ice. The boy pulls on the tow rope, oriented at 60° above horizontal, with a force of 110 N until his kid brother begins to cry. Like clockwork, his brother always cries upon reaching a speed of 2.0 m/s. The frictional coefficient is (0.20,0.15).

---

The work-energy relation tells us that:

\[ \frac{1}{2}mv_i^2 + mgh_i + \sum(F\Delta r\cos\phi) = \frac{1}{2}mv_f^2 + mgh_f \]

\[ 0 + 0 + F_{\text{rope}}(\Delta r)\cos 60 + F_{\text{surface}}(\Delta r)\cos 90 + F_{\text{kineticfriction}}(\Delta r)\cos 180 = \frac{1}{2}(30)^2 + 0 \]
\[ (110)(\Delta r)\cos 60 - F_{\text{kineticfriction}}(\Delta r) = 60 \]
\[ 55(\Delta r) - 0.15F_{\text{surface}}(\Delta r) = 60 \]

Notice that the force of the surface does no work, the force of the rope does positive work, and the force of friction does negative work. Each of these terms should make sense if you remember that work is the transfer of energy into (positive) or out of (negative) the system of interest. Also recall that in this form of the work-energy relation we conceptualize gravity as a source of potential energy, not as a force that does work.
Using the result for the force of the surface determined in the first example, \( F_{\text{surface}} = 199 \text{ N} \), gives:

\[
55(\Delta r) - 0.15 F_{\text{surface}}(\Delta r) = 60 \\
55(\Delta r) - 0.15(199)(\Delta r) = 60 \\
55(\Delta r) - 30(\Delta r) = 60 \\
25(\Delta r) = 60 \\
\Delta r = 2.4m
\]

The kid brother begins to cry after traveling 2.4 m.

### Applying Work-Energy with Gravitational Potential Energy

Let’s use the work-energy relation, with gravitational potential energy terms, to analyze the following scenario:

A 30 kg child on his 15 kg sled slides down his parents’ 10 m long, 15° above horizontal driveway after an ice storm. The coefficient of friction between the sled and the driveway is (0.10, 0.08).

Event 1: The instant before the sled begins to move.

Event 2: The instant the sled reaches the bottom of the driveway.

To calculate the gravitational energy terms, let the bottom of the driveway be zero and up positive. The coordinate system used to calculate gravitational energy does not in general have to be the same as the system you use for the rest of the problem. In fact, since the work-energy relation is a scalar equation, the other portions of the equation should not depend on your choice of coordinate system at all!

\[
\frac{1}{2}mv_f^2 + mgh_f + \sum (|F||\Delta r|\cos \phi) = \frac{1}{2}mv_i^2 + mgh_i \\
0 + 45(9.8)(10 \sin 15) + F_{\text{ice}}(10) \cos 90 + F_{\text{friction}}(10) \cos 180 = \frac{1}{2}45v_i^2 + 0 \\
1140 - 10F_{\text{friction}} = 22.5v_f^2
\]

Note:

- The only forces that could do work are the force of the ice and the force of friction, since the action of the force of gravity is already incorporated into the gravitational potential energy terms.

- The heights in the gravitational potential energy function were measured from the bottom of the driveway, with the positive direction as upward, as required. Notice that the initial height is not the same as the length of the
driveway. Since the driveway is 10 m long, at an angle of 15°, the height of the top of the driveway relative to the bottom is (10 m) sin 15°. The height at the bottom of the driveway is defined to be 0 m.

To finish the analysis we need to determine the kinetic frictional force. Since this depends on the force of the ice, apply Newton’s Second Law in the y-direction and find:

\[ \Sigma F = ma \]
\[ + F_{\text{ice}} - F_{\text{gravity}} \cos 15\degree = 45(0) \]
\[ F_{\text{ice}} - (45)(9.8) \cos 15\degree = 0 \]
\[ F_{\text{ice}} = 426N \]

\[ F_{sf} = \mu_s F_{\text{ice}} \]
\[ F_{sf} = (0.08)(426) \]
\[ F_{sf} = 34N \]

Plugging this value into the work-energy relation yields:

\[ 1140 - 10(34) = 22.5v_2^2 \]
\[ 1140 - 340 = 22.5v_2^2 \]
\[ 800 = 22.5v_2^2 \]
\[ v_2 = 5.96m/s \]

**A Two-Dimensional Collision**

Let’s try a two-dimensional collision.

At a busy intersection, an impatient driver heading south runs a red-light and collides with a delivery truck originally moving at 15 m/s west. The vehicles become entangled and the skid marks from the wreckage are at 22° south of west. The auto mass is 755 kg and the truck mass is 1250 kg.

I’ll choose:

Event 1: The instant before the car and truck collide.

Event 2: The instant the car and truck reach a common velocity
Partial free-body diagrams (top view) for both the car and the truck during this time interval are shown below.

These are only partial free-body diagrams because:

- Forces perpendicular to the earth’s surface (the force of gravity and the force of the road) are not shown.
- During a collision, the force between the colliding objects is normally much greater in magnitude than any other forces acting on the objects. Therefore we will often ignore the other forces acting on colliding objects for the duration of a collision. This approximation is termed the impulse approximation. Under the impulse approximation, the frictional forces between the car and truck and the road are ignored.

Also note that the direction of the force acting between the car and truck is unknown. The angle $\theta$ is not determined from the situation description.

Applying the impulse-momentum relation to the car and truck yields:

**Car**

<table>
<thead>
<tr>
<th>x-direction</th>
<th>y-direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$mv_i + \Sigma F(\Delta t) = mv_f$</td>
<td>$mv_i + \Sigma F(\Delta t) = mv_f$</td>
</tr>
<tr>
<td>$755(0) + F_{\text{truck on car}} \cos\theta(\Delta t) = 755(v_2 \cos 22)$</td>
<td>$755(v_{\text{car}}) - F_{\text{truck on car}} \sin\theta(\Delta t) = 755(v_2 \sin 22)$</td>
</tr>
</tbody>
</table>

**Truck**

<table>
<thead>
<tr>
<th>x-direction</th>
<th>y-direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$mv_i + \Sigma F(\Delta t) = mv_f$</td>
<td>$mv_i + \Sigma F(\Delta t) = mv_f$</td>
</tr>
<tr>
<td>$1250(15) - F_{\text{car on truck}} \cos\theta(\Delta t) = 1250(v_2 \cos 22)$</td>
<td>$1250(0) + F_{\text{car on truck}} \sin\theta(\Delta t) = 1250(v_2 \sin 22)$</td>
</tr>
</tbody>
</table>
Since the magnitude of the force on the car due to the truck and the force on the truck due to
the car are equal, when the x-equations for the car and truck are added, the impulses cancel!

\[ \text{x-direction} \]

\[
755(0) + 1250(15) = 755(v_2 \cos 22) + 1250(v_2 \cos 22) \\
18750 = 2005(v_2 \cos 22) \\
v_2 = 10.1 m/s
\]

This is the speed of the wreckage immediately after the collision. Note that this is exactly the
same equation we would have written if we had considered the system of the car and truck
right from the start.

Adding the two y-equations yields:

\[ \text{y-direction} \]

\[
755(v_{\text{car}}) + 1250(0) = 755(v_2 \sin 22) + 1250(v_2 \sin 22) \\
755(v_{\text{car}}) = 2005(10.1 \sin 22) \\
v_{\text{car}} = 10.0 m/s
\]

This is the speed of the car immediately before colliding with the truck.
Conservation Laws

Activities
Below are six different directions in which a baseball can be thrown. In all cases the baseball is thrown at the same initial speed from the same height above the ground. Assume the effects of air resistance are negligible.

a. Rank these baseballs on the basis of their horizontal speed the instant before they hit the ground.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest _____ The ranking cannot be determined based on the information provided.

b. Rank these baseballs on the basis of their vertical speed the instant before they hit the ground.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest _____ The ranking cannot be determined based on the information provided.

c. Rank these baseballs on the basis of their speed the instant before they hit the ground.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest _____ The ranking cannot be determined based on the information provided.

Explain the reason for your rankings:
Below are six different directions and heights from which a baseball can be thrown. In all cases the baseball is thrown at the same speed, \( v \). Assume the effects of air resistance are negligible.

\[
\begin{align*}
&\text{H} & \text{\( \theta \)} \\
A & 10 \text{ m} & 30^0 \\
B & 10 \text{ m} & 0^0 \\
C & 10 \text{ m} & 60^0 \\
D & 20 \text{ m} & 0^0 \\
E & 15 \text{ m} & 45^0 \\
F & 5 \text{ m} & 90^0 \\
\end{align*}
\]

Rank these baseballs on the basis of their speed the instant before they hit the ground.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest _____

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
A crate is released from rest along an inclined surface. The mass of the crate and the angle of the incline vary. The frictional coefficients between the crates and the surfaces are identical and so small that the effect of friction is negligible. All crates are released from the same vertical height, $H$, above the bottom of the incline.

<p>| | | |</p>
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>m</td>
<td>$\theta$</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>10 kg</td>
<td>30°</td>
</tr>
<tr>
<td>B</td>
<td>20 kg</td>
<td>15°</td>
</tr>
<tr>
<td>C</td>
<td>10 kg</td>
<td>60°</td>
</tr>
<tr>
<td>D</td>
<td>20 kg</td>
<td>60°</td>
</tr>
<tr>
<td>E</td>
<td>15 kg</td>
<td>45°</td>
</tr>
<tr>
<td>F</td>
<td>5 kg</td>
<td>85°</td>
</tr>
</tbody>
</table>

a. Rank these scenarios on the basis of the kinetic energy of the crate the instant it reaches the bottom of the incline.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest _____

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

b. Rank these scenarios on the basis of the speed of the crate the instant it reaches the bottom of the incline.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest _____

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
A crate is released from rest along an inclined surface. The mass of the crate and the angle of the incline vary. The frictional coefficients between the crates and the surfaces are identical and so small that the effect of friction is negligible. All crates are released from the same distance, D, along the incline.

<table>
<thead>
<tr>
<th></th>
<th>m (kg)</th>
<th>θ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>B</td>
<td>20</td>
<td>15</td>
</tr>
<tr>
<td>C</td>
<td>10</td>
<td>60</td>
</tr>
<tr>
<td>D</td>
<td>20</td>
<td>60</td>
</tr>
<tr>
<td>E</td>
<td>15</td>
<td>45</td>
</tr>
<tr>
<td>F</td>
<td>5</td>
<td>85</td>
</tr>
</tbody>
</table>

a. Rank these scenarios on the basis of the kinetic energy of the crate the instant it reaches the bottom of the incline.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest _____

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

b. Rank these scenarios on the basis of the speed of the crate the instant it reaches the bottom of the incline.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest _____

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
Below are bird’s-eye views of six automobile crashes an instant before they occur. The automobiles have different masses and velocities. All automobiles will remain joined together after the impact and skid to rest. Rank these automobile crashes on the basis of the angle at which the wreckage skids. Let 0° be the angle oriented directly toward the right and measure angles counterclockwise from 0°.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest _____

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
Below are bird's-eye views of six automobile crashes an instant before they occur. The automobiles have different masses and velocities. All automobiles will remain joined together after the impact and skid to rest at the same angle, as measured from a line oriented directly toward the right. Rank these scenarios on the basis of the initial speed of the auto traveling toward the top of the page.

A

B

C

D

E

F

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest _____

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
For each of the collisions illustrated below, sketch a graph of the momentum of asteroid A, the momentum of asteroid B, and the total momentum in the system of the two asteroids. Sketch the horizontal and vertical momentum separately. Begin your graph before the collision takes place and continue it after the collision is over. Use a consistent scale on all graphs.

a. The two asteroids remain joined together after the collision.

b. The two asteroids remain joined together after the collision.
For each of the collisions illustrated below, sketch a graph of the momentum of asteroid A, the momentum of asteroid B, and the total momentum in the system of the two asteroids. Sketch the horizontal and vertical momentum separately. Begin your graph before the collision takes place and continue it after the collision is over. The asteroids’ initial velocities are both oriented at the same angle from horizontal. Use a consistent scale on all graphs.

a. The two asteroids remain joined together after the collision.

b. The two asteroids remain joined together after the collision.
For each of the collisions illustrated below, sketch a graph of the momentum of asteroid A, the momentum of asteroid B, and the total momentum in the system of the two asteroids. Sketch the horizontal and vertical momentum separately. Begin your graph before the collision takes place and continue it after the collision is over.

a. The two asteroids remain joined together after the collision and move directly toward the top of the page.

b. The two asteroids remain joined together after the collision and move directly toward the right. The asteroids’ initial velocities are both oriented at the same angle from horizontal.
For each of the explosions illustrated below, sketch a graph of the momentum of fragment A, B, and C, and the total momentum in the system of the three asteroids. Sketch the horizontal and vertical momentum separately. Begin your graph before the explosion takes place and continue it as the fragments move apart. The exploding egg is initially at rest.

a. Fragment A moves horizontally and fragments B and C move at the same angle from horizontal after the explosion.

b. Fragment B moves vertically and fragments A and C move at the same angle from vertical after the explosion.
For each of the scenarios described below, indicate the amount of kinetic energy and gravitational potential energy of the object at each of the events listed. Use a consistent scale throughout both motions. Set ground level as the zero-point of gravitational potential energy.

a. A baseball is thrown at 30 m/s at an angle of 30° above horizontal over level ground.

b. A baseball is thrown at 30 m/s at an angle of 60° above horizontal over level ground.
For each of the scenarios described below, indicate the amount of kinetic energy and gravitational potential energy of the object at each of the events listed. Use a consistent scale throughout both motions.

a. A 60 kg skier starts from rest at the top of a 100 m, 25° slope. He doesn’t push with his poles because he’s afraid of going too fast. Set the bottom of the slope as the zero-point of gravitational potential energy.

b. A 60 kg skier starts from rest at the top of a 100 m, 25° slope. He doesn’t push with his poles because he’s afraid of going too fast. Set the top of the slope as the zero-point of gravitational potential energy.
For each of the scenarios described below, indicate the amount of kinetic energy and gravitational potential energy of each object at each of the events listed. Use a consistent scale throughout both motions. Set the initial positions of the skier and block as the zero-points of gravitational potential energy.

a. In a horizontal skiing device, the skier begins from rest 35 m from the end of the skiing run. The ballast block has a mass of 50 kg and the skier has a mass of 75 kg. The coefficient of friction is extremely small.

b. In an inclined skiing device, the skier begins from rest 35 m from the end of the 20° above horizontal inclined skiing run. The ballast block has a mass of 50 kg and the skier has a mass of 75 kg. The coefficient of friction is extremely small.
A girl pulls a 35 kg sled, including the mass of her kid sister, along ice. The girl pulls on the tow rope, oriented at 40° above horizontal, with a force of 120 N until her sister begins to cry. Like clockwork, her sister always cries upon reaching a speed of 3.0 m/s. The frictional coefficient is (0.10, 0.08).

**Free-Body Diagram**

**Mathematical Analysis**

- Event 1:
- Event 2:

*a. How far has the sled moved before the little sister begins to cry?*

*b. What is the elapsed time before the little sister begins to cry?*
A boy pulls a 30 kg sled, including the mass of his kid brother, along ice. The boy pulls on the tow rope, oriented at 60° above horizontal, with a force of 110 N for 3.0 s. At the end of the 3.0 s pull, his kid brother begins to cry. The frictional coefficient is (0.20, 0.15).

**Free-Body Diagram**

**Mathematical Analysis**

Event 1:

Event 2:

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*a. How fast is the sled moving before the little brother begins to cry?*

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*b. How far has the sled moved before the little brother begins to cry?*
Starting from rest, a girl can pull a sled, carrying her kid brother, 20 m in 8 s. The girl pulls on the tow rope, oriented at \(30^\circ\) above horizontal, with a force of 90 N. The frictional coefficient is \((0.15, 0.10)\).

**Free-Body Diagram**

![Free-Body Diagram](image)

**Mathematical Analysis**

Event 1:

Event 2:
A 100 kg bicycle and rider initially move at 16 m/s up a 15° hill. The rider slams on the brakes and skids to rest. The coefficient of friction is (0.8, 0.7).

Free-Body Diagram

Mathematical Analysis

Event 1:

Event 2:

a. How far does the bike skid?

b. What is the elapsed time before the bike stops skidding?
A 60 kg skier starts from rest at the top of a 100 m, 25° slope. He doesn’t push with his poles because he’s afraid of going too fast. The frictional coefficient is (0.10,0.05).

Free-Body Diagram

Mathematical Analysis

Event 1:
Event 2:
A 70 kg snowboarder starts from rest at the top of a 270 m, 20° slope. At the bottom of the hill she’s moving at 33 m/s.

Free-Body Diagram

Mathematical Analysis

Event 1:

Event 2:
The device at right guarantees all the excitement of skiing without the need for hills. The skier begins from rest 35 m from the brick wall. The block has a mass of 50 kg and the skier has a mass of 75 kg. The coefficient of friction is (0.15,0.13).

Free-Body Diagrams

Mathematical Analysis

Event 1:
Event 2:
The device at right allows novices to ski downhill at reduced speeds. The block has a mass of 10 kg and the skier has a mass of 80 kg. The coefficient of friction is \((0.08,0.07)\). The skier starts from rest at the top of a 30 m, 20° slope.

**Free-Body Diagrams**

- **skier**
- **block**

**Mathematical Analysis**

- Event 1:
- Event 2:
The device at right allows you to ski uphill (until you smash into the pulley). The ballast block has a mass of 60 kg and the skier has a mass of 70 kg. The coefficient of friction is (0.1, 0.09). The ramp is inclined at 20° above horizontal and the pulley is 45 m away.

Free-Body Diagrams

Mathematical Analysis

Event 1:

Event 2:
Two identical 800 kg automobiles, one moving east at 10 m/s and the other moving north at 15 m/s, collide. After the collision they remain joined together and move with a common velocity.

Free-Body Diagrams

Mathematical Analysis

Event 1:
Event 2:
Two identical 750 kg automobiles, one moving east at 10 m/s and the other moving north, collide. After the collision they remain joined together and move with a common velocity. The wreckage skids at $30^\circ$ north of east.

**Free-Body Diagrams**

**Mathematical Analysis**

Event 1:
Event 2:
In a demolition derby, a 700 kg Audi is traveling at 15 m/s 30° north of east. An 800 kg BMW is traveling at 5.0 m/s south. They collide. After the collision, the Audi is redirected to 10° north of east and the BMW is redirected to 40° east of south.

**Free-Body Diagrams**

**Mathematical Analysis**

Event 1:

Event 2:
In a demolition derby, a 600 kg Audi is traveling at 15 m/s 30° west of south. A 700 kg BMW is traveling at 10 m/s 40° north of east. They collide. After the collision, the Audi is redirected to 20° north of west and the BMW is redirected to 50° south of east.

Free-Body Diagrams

Mathematical Analysis

Event 1:
Event 2:
A boy pulls an initially stationary sled of mass m (including the mass of the strange neighborhood kid riding the sled) along a level surface. He exerts a force of magnitude $F$ at an angle of $\theta$ above the horizontal. Determine the velocity ($v$) of the sled as a function of the distance pulled ($d$), the appropriate coefficient of friction between the sled and the surface, $m$, $F$, $\theta$, and $g$.

**Questions**

If $d = 0$ m, what should $v$ equal? Does your function agree with this observation?

If $m = 0$ kg, what should $v$ equal? Does your function agree with this observation?

If $F = 0$ N, what should $v$ equal? Does your function agree with this observation?
A skier of mass \( m \) starts from rest at the top of a slope of length \( D \) inclined at \( \theta \) above horizontal. She does not push with her poles. Determine the speed of the skier at the bottom of the slope (\( v \)) as a function of the appropriate coefficient of friction between the skies and the snow, \( D \), \( m \), \( \theta \), and \( g \).

**Free-Body Diagram**

**Mathematical Analysis**

Event 1:

Event 2:

**Questions**

If \( g = 0 \, \text{m/s}^2 \), what should \( v \) equal? Does your function agree with this observation?

If \( \theta = 0^\circ \), what should \( v \) equal? Does your function agree with this observation?

If the \( D \) is doubled, what will happen to \( v \)?
The driver of an automobile of mass \( m \), traveling down an incline of angle \( \theta \), suddenly sees an obstacle blocking her lane. Ignoring her reaction time, determine the time elapsed (\( T \)) before the car skids to a stop as a function of the initial velocity (\( v \)), the appropriate coefficient of friction between the tires and the road, \( m \), \( \theta \), and \( g \).

**Free-Body Diagram**

**Mathematical Analysis**

Event 1:

Event 2:

**Questions**

If \( \mu = 0 \), what should \( T \) equal? Does your function agree with this observation?

If \( m \) is doubled, what will happen to \( T \)?

If \( v \) is doubled, what will happen to \( T \)?

Is there a maximum angle above which the car will not stop? If so, determine an expression for this angle.
The driver of an automobile of mass $m$, traveling down an incline of angle $\theta$, suddenly sees an obstacle blocking her lane. Ignoring her reaction time, determine the distance ($D$) the car skids before stopping as a function of the initial velocity ($v$), the appropriate coefficient of friction between the tires and the road, $m$, $\theta$, and $g$.

**Free-Body Diagram**

**Mathematical Analysis**

Event 1:

Event 2:

**Questions**

If $\mu = 0$, what should $D$ equal? Does your function agree with this observation?

If $m$ is doubled, what will happen to $D$?

If $v$ is doubled, what will happen to $D$?

Is there a maximum angle above which the car will not stop? If so, determine an expression for this angle.
Two identical automobiles, one moving east at $v_E$ and the other moving north at $v_N$, collide. After the collision they remain joined together and move with a common velocity. Determine the angle at which the wreckage skids ($\theta$), measured counterclockwise from east, as a function of $v_E$ and $v_N$.

**Free-Body Diagrams**

**Mathematical Analysis**

Event 1:

Event 2:

Questions

If $v_E = \infty$, what should $\theta$ equal? Does your function agree with this observation?

If $v_E = v_N$ what should $\theta$ equal? Does your function agree with this observation?