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Spiral Physics

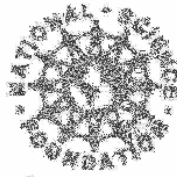
Applications

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The Ideal Gas

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This project was supported, in part, by the National Science Foundation. Opinions expressed are those of the author and not necessarily those of the Foundation.





Model Specifics

The Ideal Gas

In this section, we will investigate a new, mechanical model that can be viewed as an application of the models previously studied. Although we will take a step back in complexity and examine a particle, rather than a rigid-body, model, to compensate for this simplification we will not look at one particle, or two particles, but rather *many* particles. "Many" in this case means approximately 10^{23} !

Since there are *so* many particles, we will no longer be interested in the motion of any one individual particle, but rather in the average motions of the entire group of particles. We will try to use our understanding of mechanics, applied to individual *microscopic* particles, to learn about the *macroscopic* properties of the entire group.

To simplify matters, we will assume that the particles do not interact except via elastic collisions with each other and collisions with the walls of the container in which they are contained. Thus, there are no non-contact interactions, such as the force of gravity, nor contact interactions, such as connecting ropes, springs, etc, between the particles.

If we imagine our collection of particles to represent individual atoms in a gas, this model is referred to as the *ideal gas model*. The study of the ideal gas forms the entrance into the field of physics known as thermodynamics.

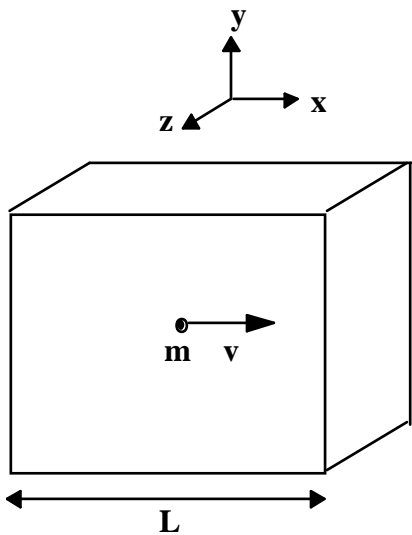
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The Ideal Gas

Concepts and Principles

Applying Newton's Second Law

Although the ultimate goal is to study a collection of a large number of particles, we'll start out by looking at just a single particle. Imagine a particle of mass m and speed v confined inside an elastic cube of edge length L .



Since we're imagining, imagine the particle to be moving in the +x-direction, perpendicular to a face of the cube. The particle will strike the wall and bounce off. Let's apply Newton's second law to the particle during the time interval in which it is in contact with the wall, $\Delta t_{\text{collision}}$:

$$\Sigma F = ma$$

$$F_{\text{average}} = m\left(\frac{\Delta v}{\Delta t_{\text{collision}}}\right)$$

Notice that we have replaced the instantaneous force acting on the particle with the average force over the time interval of interest. Since the wall is perfectly elastic, the collision should just reverse the direction of the particle's speed.

$$F_{\text{average}} = m\left(\frac{v_f - v_i}{\Delta t_{\text{collision}}}\right)$$

$$F_{\text{average}} = m\left(\frac{(-v) - (v)}{\Delta t_{\text{collision}}}\right)$$

$$F_{\text{average}} = \frac{-2mv}{\Delta t_{\text{collision}}}$$

This is the average force on the particle. The average force on the wall is equal in magnitude but opposite in direction.

$$F_{\text{averageonwall}} = \frac{2mv}{\Delta t_{\text{collision}}}$$

This is the average force on the wall during the relatively small time interval of the collision. There are long lulls (when the particle is moving toward the other wall and then returning) that the average force on the wall is zero. If we want the average force on the wall during the entire motion of the particle (and we do) we have to divide not by $\Delta t_{\text{collision}}$ but by $\Delta t_{\text{round-trip}}$, the time to make a complete round-trip journey. Thus, the average force on the wall during the entire journey of the particle is:

$$F_{\text{averageonwall}} = \frac{2mv}{\Delta t_{\text{round-trip}}}$$

Since we know the speed of the particle, we know $\Delta t_{\text{round-trip}}$. A total distance traveled of $2L$, at a speed of v , takes a time of:

$$\Delta t_{\text{round-trip}} = \frac{2L}{v}$$

so

$$F_{\text{averageonwall}} = \frac{2mv}{\left(\frac{2L}{v}\right)}$$

$$F_{\text{averageonwall}} = \frac{mv^2}{L}$$

Thus, we have determined the average force a *single* particle of mass m and speed v would exert on the wall of a cube of edge length L . The question remains, however, what would this average force be if we had N particles?

To generalize our relation we must make two adjustments. First, if we had N identical particles, all moving in an identical manner, it would seem obvious that the average force on the wall would be N times greater,

$$F_{\text{averageonwall}} = \frac{Nmv^2}{L}$$

This would be true, however, only if all N particles were moving with the same speed in the same direction, in this case the x direction.

$$F_{\text{averageonwall}} = \frac{Nmv_x^2}{L}$$

What if the particles were moving in all directions at many possible speeds? *On average*, there is no reason why the particles should be going faster in the x than the y or the z direction. Thus,

$$(v_x^2)_{average} = (v_y^2)_{average} = (v_z^2)_{average}$$

Since

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

we will assume that

$$(v^2)_{average} = (v_x^2)_{average} + (v_y^2)_{average} + (v_z^2)_{average}$$

$$(v^2)_{average} = (v_x^2)_{average} + (v_x^2)_{average} + (v_x^2)_{average}$$

$$(v^2)_{average} = 3(v_x^2)_{average}$$

$$(v_x^2)_{average} = \frac{1}{3}(v^2)_{average}$$

So, averaging over all the various velocities leads to:

$$F_{averageonwall} = \frac{Nm(v^2)_{average}}{3L}$$

Let's try to remember that the force and the squared speed are averages, yet state the relation as:

$$F = \frac{Nmv^2}{3L}$$

It would be nice to have a relationship that does not explicitly depend on the length of the container. We can accomplish this by defining the *pressure* (p) of the gas as the average force exerted by the gas on one square meter of its container. Thus, pressure is measured in Newtons per square meter, which is given the formal SI name of *pascal*, abbreviated Pa.

If we take the above expression and divide it by the surface area of the wall, L^2 , the resulting ratio will be the pressure of the gas on the wall of the container.

$$F = \frac{Nmv^2}{3L}$$
$$\frac{F}{L^2} = \frac{Nmv^2}{3L^3}$$
$$p = \frac{Nmv^2}{3L^3}$$
$$p = \frac{Nmv^2}{3V}$$

with L^3 replaced by V , the volume of the container.

We now have a relationship that combines macroscopic quantities, such as the pressure and volume of the gas, and microscopic quantities, such as the mass and average squared velocity of individual particles.

To complete the macroscopic description, I will make a radical definition: I hereby define the average kinetic energy of an individual particle to be proportional to a quantity I will call *temperature* (T).¹

$$\frac{1}{2}mv^2 \propto T$$

Thus, temperature is a macroscopic measurement of a microscopic quantity; an individual particle's average kinetic energy. Selecting a proportionality constant so that the resulting relationship is as simple as possible leads me to select " $3/2 k$ " as the constant. The " $3/2$ " makes the resulting relation simple and the " k " is in honor of Ludwig Boltzmann and is called the Boltzmann constant.²

$$\frac{1}{2}mv^2 = \frac{3}{2}kT$$
$$mv^2 = 3kT$$

¹ We will measure temperature in kelvin, abbreviated K.

² The Boltzmann constant is experimentally determined to be 1.38×10^{-23} joules/kelvin.

Therefore our relationship between macroscopic variables becomes

$$p = \frac{Nmv^2}{3V}$$
$$p = \frac{N(3kT)}{3V}$$
$$p = \frac{NkT}{V}$$
$$pV = NkT$$

This relation is termed the *equation of state* for an ideal gas because it forms a relationship between the parameters that define the state the gas is in; its pressure, volume and temperature.

An alternative form of the same relation, written in terms of the number of moles of particles³ (n) rather than number of particles (N) is

$$pV = nRT$$

By comparing the two relations you should notice that R, the *universal gas constant*, is equal to the product of the Boltzmann constant and the number of particles in a mole. Thus, R = 8.31 joules/kelvin.

Applying the Work-Energy Relation

The equation of state relates the *state variables* of a gas: pressure, volume and temperature. If you know the value of two of these variables, you can always determine the third. However, the relation says nothing about the *process* by which the gas arrived at that state. The easiest way to investigate processes is through the application of the work-energy relation.

In general terms, the work-energy relation states that the energy of a particle is changed through the application of work. Schematically,

$$\text{initial energy} + \text{work} = \text{final energy}$$

Thus, in general terms, work is the transfer of energy to a particle. We could state the work-energy relation in these more general terms.

$$\text{initial energy} + \text{energy transfer} = \text{final energy}$$

³ 6.02 x 10²³ particles comprise one mole of particles.

Since our collection of particles is non-interacting, there is no gravitational or elastic potential energy present. The only form of energy present is kinetic.

$$\textit{initial kinetic energy} + \textit{energy transfer} = \textit{final kinetic energy}$$

This form of the work-energy relation must hold for each and every particle. If we apply this result to all of the particles in the gas, and sum the results, we arrive at

$$\textit{total initial KE of all particles} + \textit{total energy transfer to gas} = \textit{total final KE of all particles}$$

The total kinetic energy of all of the particles in a gas is termed the *internal energy* (U_{internal}) of the gas. Thus,

$$U_{\text{internal},i} + \textit{total energy transfer to gas} = U_{\text{internal},f}$$

with

$$U_{\text{internal}} = N\left(\frac{1}{2}mv^2\right)$$

$$U_{\text{internal}} = N\left(\frac{3}{2}kT\right)$$

$$U_{\text{internal}} = \frac{3}{2}NkT$$

or

$$U_{\text{internal}} = \frac{3}{2}nRT$$

Now comes the tricky part. What is the total energy transfer to the gas? Remember that the energy transfer to each particle is due to the application of forces external to that particle. Since the particles do not interact with each other, except through elastic collisions, all the energy transfer must be done through collisions of the particles with the walls of the container. This transfer of energy to the gas is via two, distinct methods.

Macroscopic Energy Transfer

First imagine a collision of a particle with a stationary wall. An analogy to this situation would be a tennis ball bouncing off of a stationary tennis racket. The tennis ball rebounds off of the racket with approximately the same speed, and hence the same energy, as it had when it struck the racket. Thus, there is no energy transfer to the ball during this type of collision.

Now imagine a tennis ball striking a moving racket. If the racket is moving toward the ball, the ball will rebound off the racket with a greatly increased speed. Hence, there is energy transfer *to* the ball during this collision. If the racket is moving away from the ball, the ball will rebound with a decreased speed. Hence, there is energy transfer *from* the ball during this collision.

From this analogy, we can see that energy can be transferred to or from the gas of particles if the walls of the container are in motion. If the walls of the container are in motion, then the volume of the container must be changing. Thus, if the volume of the container is changing, an energy transfer is taking place. This energy transfer is termed *macroscopic* because the mechanism causing it is the motion of a macroscopic object, a container wall.

Since the container walls are moved by the application of forces acting over distances, this energy transfer is the familiar energy transfer termed *work*. In fact, from the definition of work,

$$W = |F||\Delta r| \cos \phi$$

with F the average force acting on the container wall and Δr the distance the container wall moves.

The force acting on the container wall can be written in terms of the pressure on the wall and the area of the wall, and let Δr be the distance the container wall moves in the direction of the force:

$$W = pA(\Delta r)$$

A change in wall position, Δr , leads to a change in container volume, ΔV , given by

$$\Delta V = A(\Delta r)$$

Thus,

$$W = p(\Delta V)$$

This is the work done *on* the container *by* the gas. Since we are interested in the energy transfer to the gas, we need the work done *on* the gas *by* the container, which is

$$W = -p(\Delta V)$$

Work is the macroscopic transfer of energy to the gas by means of changes in gas volume.

Microscopic Energy Transfer

Now let's look more closely at the collision of a gas particle with the container wall. Realistically, the container wall is composed of a huge number of particles. The container wall also has a definite temperature, implying that the particles comprising the wall have average kinetic energies proportional to this temperature. Thus, *even if the wall is macroscopically stationary, the individual particles comprising the wall are in motion.* Therefore, the collision of a gas particle is not really with a stationary wall but rather with a microscopically moving wall particle! In addition, since we already know that energy can be transferred during an elastic collision between two moving objects, we should realize that energy can be transferred even when the walls are macroscopically stationary. This *microscopic* transfer of energy is termed *heat*.

However, let's not get carried away. Granted, the wall particles are in motion, but aren't the collisions between gas particles and wall particles equally likely to occur when the wall particles are moving toward the gas particles (transfer of energy to the gas) as when the wall particles are moving away from the gas particles (transfer of energy from the gas)? If this is true, the net energy flow should be zero!

The above argument *is* true, assuming the colliding particles have the same kinetic energy as the wall particles. The argument is simply a complicated way of saying that if the wall and the gas particles have the same kinetic energy, and are therefore at the same temperature, no net energy will be transferred from the wall to the gas. There will be no transfer of energy via heat if both objects are at the same temperature.

But what if the temperature of the wall is higher than that of the gas? This means that the wall particles have more kinetic energy than the gas particles. Although you will still have some collisions in which energy is transferred from the gas (when the wall particles are moving away from the gas particles), the amount of energy transferred to the gas (when the wall particles are moving toward the gas particles) will be greater. Overall, the particles with more kinetic energy will transfer energy to the particles with less kinetic energy. Hence, energy will flow from high kinetic energy particles to low kinetic energy particles, or, put more simply, from high temperature to low temperature. Again, this microscopic flow of energy from high temperature to low temperature particles is called heat. Physicists use the symbol Q to represent the amount of energy transferred via heat.

Thus, our application of the work-energy relation to an ideal gas leaves us with

$$U_{\text{internal, i}} + \text{total energy transfer to gas} = U_{\text{internal, f}}$$

$$U_{\text{internal, i}} + W + Q = U_{\text{internal, f}}$$

This relationship, often referred to as the First Law of Thermodynamics, states that the internal energy of a gas can be changed through energy transfer to the gas via work or heat.

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The Ideal Gas

Analysis Tools

A Constant Pressure Process

An approximately spherical, 9.4 cm radius helium balloon is brought from a chilly room (15 °C) into a warm room (20 °C).

To analyze this situation, we should first carefully determine and define the sequence of events that take place. At each of these instants, let's tabulate what we know about the state of the helium gas. Then, we can analyze the process that leads from one state to another.

Event 1: The instant before the balloon is brought into the warmer room	Event 2: The instant the balloon stops expanding
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$$p_1 =$$

$$p_2 =$$

$$V_1 = \frac{4}{3} \pi (0.094 \text{ m})^3$$

$$V_2 =$$

$$T_1 = 15 \text{ }^\circ\text{C} = 288 \text{ K}$$

$$T_2 = 20 \text{ }^\circ\text{C} = 293 \text{ K}$$

Process:

$$W_{12} =$$

$$Q_{12} =$$

The situation describes a gas that undergoes a process taking it from one state to another. Therefore, at each state I have tabulated the value (if know) of each of the state variables. Between the states I have indicated a location for the recording of the process undertaken as well as the energy transfer during the process, via work or heat.

Do I know anything else about the initial state of the gas in the balloon? The force that the helium gas exerts on the inner surface of the balloon must be equal in magnitude to the force that the outside air exerts on the outer surface of the balloon plus the additional force inward due to the elastic nature of the balloon. If I ignore this additional force due to the elastic balloon, the pressure of the helium must be equal in magnitude to the outside air pressure. Average air pressure on the surface of the earth, referred to as an atmosphere (atm), is

$$1 \text{ atm} = 1.01325 \times 10^5 \text{ Pa}$$

In addition, since the outside air pressure doesn't change, neither does the pressure of the helium gas. Therefore, the process is at constant pressure.

Event 1: The instant before the balloon is brought into the warmer room	Event 2: The instant the balloon stops expanding
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$$p_1 = 1.01 \times 10^5 \text{ Pa}$$

$$p_2 = 1.01 \times 10^5 \text{ Pa}$$

$$V_1 = 3.48 \times 10^{-3} \text{ m}^3$$

$$V_2 =$$

$$T_1 = 288 \text{ K}$$

$$T_2 = 293 \text{ K}$$

Process: Constant Pressure

$$W_{12} =$$

$$Q_{12} =$$

The next step is to apply the equation of state to the two states of the gas:

$$p_1 V_1 = nRT_1$$

$$p_2 V_2 = nRT_2$$

$$(1.01 \times 10^5 \text{ Pa})(3.48 \times 10^{-3} \text{ m}^3) = n(8.31 \text{ J/K})(288 \text{ K}) \quad (1.01 \times 10^5 \text{ Pa}) V_2 = (0.147)(8.31 \text{ J/K})(293 \text{ K})$$

$$n = 0.147 \text{ moles}$$

$$V_2 = 3.54 \times 10^{-3} \text{ m}^3$$

Thus, the final radius of the balloon is:

$$r_2 = 9.46 \text{ cm}$$

We have completely defined the initial and final states of the helium gas in the balloon, but what about the process that took the balloon from the first state to the second state? From the First Law of Thermodynamics,

$$U_{\text{internal, i}} + W + Q = U_{\text{internal, f}}$$

$$\frac{3}{2} nRT_1 + (- p(\Delta V)) + Q = \frac{3}{2} nRT_2$$

Since the process is at constant pressure,

$$W = - p(\Delta V)$$

$$W = - p (V_2 - V_1)$$

$$W = - (1.01 \times 10^5 \text{ Pa})(3.54 - 3.48) \times 10^{-3} \text{ m}^3$$

$$W = - 6.06 \text{ J}$$

Thus,

$$\frac{3}{2} nRT_1 + (- p(\Delta V)) + Q = \frac{3}{2} nRT_2$$

$$\frac{3}{2} (0.147)(8.31)(288) - 6.06 + Q = \frac{3}{2} (0.147)(8.31)(293)$$

$$Q = 15.2 \text{ J}$$

During the constant pressure process, 15.2 J of energy were transferred to the gas via heat (from the 20 °C room) and 6.06 J of energy were transferred out of the gas via the work done by the gas in expanding the balloon.

A Constant Volume Process

A miniature (1.1 m^3), robotic submarine is loaded with scientific apparatus at the ocean's surface, sealed, and programmed to dive from the surface (15°C) to a depth of 3000 m (3°C).

When the submarine is open at the ocean's surface, the air inside the sub is at a pressure of 1 atm. The sub is then sealed so that no air can leak in or out. Assume the sub is made of strong material so that its volume does not change during its descent.

Event 1: The instant the submarine is sealed at the surface	Event 2: The instant the submarine achieves temperature equilibrium at a depth of 3000 m
$p_1 = 1.01 \times 10^5 \text{ Pa}$	$p_2 =$
$V_1 = 1.1 \text{ m}^3$	$V_2 = 1.1 \text{ m}^3$
$T_1 = 288 \text{ K}$	$T_2 = 276 \text{ K}$

Process: Constant Volume

$$W_{12} =$$

$$Q_{12} =$$

The next step is to apply the equation of state to the two states of the gas:

$$p_1 V_1 = nRT_1$$

$$(1.01 \times 10^5 \text{ Pa})(1.1 \text{ m}^3) = n(8.31 \text{ J/K})(288 \text{ K})$$

$$n = 46.4 \text{ moles}$$

$$p_2 V_2 = nRT_2$$

$$p_2 (1.1 \text{ m}^3) = (46.4)(8.31 \text{ J/K})(276 \text{ K})$$

$$p_2 = 0.968 \times 10^5 \text{ Pa}$$

We have completely defined the initial and final states of the gas in the sub, but what about the process that took the gas from the first state to the second state? From the First Law of Thermodynamics,

$$U_{\text{internal}, i} + W + Q = U_{\text{internal}, f}$$

$$\frac{3}{2} nRT_1 + (-p(\Delta V)) + Q = \frac{3}{2} nRT_2$$

Since the process is at constant volume, no energy transfer can take place through work. Thus,

$$\frac{3}{2} nRT_1 + 0 + Q = \frac{3}{2} nRT_2$$

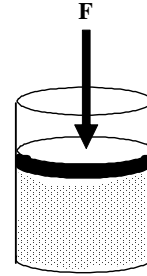
$$\frac{3}{2} (46.4)(8.31)(288) + Q = \frac{3}{2} (46.4)(8.31)(276)$$

$$Q = -6940 \text{ J}$$

During the sub's descent, 6940 J of energy were transferred out of the gas into the surrounding ocean water via heat.

A Constant Temperature Process

The piston at right is immersed in an ice-water bath. With a 100 N force applied to the circular, 2 cm radius piston head, the volume of gas in the piston is 200 cm³. The external force is slowly reduced to zero, leaving only atmospheric pressure pressing on the piston head.



Since the piston is immersed in ice water, the initial temperature of the gas is 0 °C. If the external force is removed slowly, the temperature of the gas should always remain in equilibrium with the ice water bath. Hence, the final temperature of the gas is also 0 °C. Note that the initial pressure of the gas is equal to the sum of atmospheric pressure plus the pressure due to the applied force.

Event 1: The instant before the force is decreased

$$p_1 = 1.01 \times 10^5 \text{ Pa} + 100 \text{ N}/(\pi(0.02 \text{ m})^2)$$

$$p_1 = 1.81 \times 10^5 \text{ Pa}$$

$$V_1 = 200 \text{ cm}^3 = 2 \times 10^{-4} \text{ m}^3$$

$$T_1 = 273 \text{ K}$$

Event 2: The instant the applied force reaches 0 N

$$p_2 = 1.01 \times 10^5 \text{ Pa}$$

$$V_2 =$$

$$T_2 = 273 \text{ K}$$

Process: Constant Temperature

$$W_{12} =$$

$$Q_{12} =$$

The next step is to apply the equation of state to the two states of the gas:

$$p_1 V_1 = nRT_1$$

$$(1.81 \times 10^5 \text{ Pa})(2 \times 10^{-4} \text{ m}^3) = n(8.31 \text{ J/K})(273 \text{ K})$$

$$n = 0.0159 \text{ moles}$$

$$p_2 V_2 = nRT_2$$

$$(1.01 \times 10^5) V_2 = (0.0159)(8.31 \text{ J/K})(273 \text{ K})$$

$$V_2 = 3.57 \times 10^{-4} \text{ m}^3$$

We have completely defined the initial and final states of the gas in the piston, but what about the process that took the gas from the first state to the second state? From the First Law of Thermodynamics,

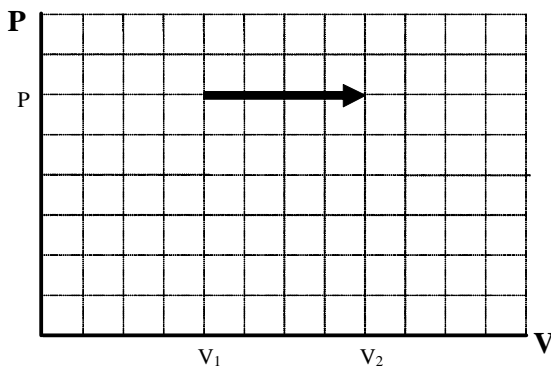
$$U_{\text{internal, i}} + W + Q = U_{\text{internal, f}}$$

$$\frac{3}{2} nRT_1 + (- p(\Delta V)) + Q = \frac{3}{2} nRT_2$$

Let's examine the work done on the gas.

$$W = - p(\Delta V)$$

This is going to get a little tricky because the pressure is changing during the process. In other words, what value for pressure goes into the above equation, p_1 or p_2 ? To resolve this dilemma, let's re-examine a situation that was not as tricky, the constant pressure process. We will examine this process on a pressure vs. volume graph.



On the graph at left, a constant pressure process is represented by a horizontal line. We also know that the work done on the gas in a constant pressure process is

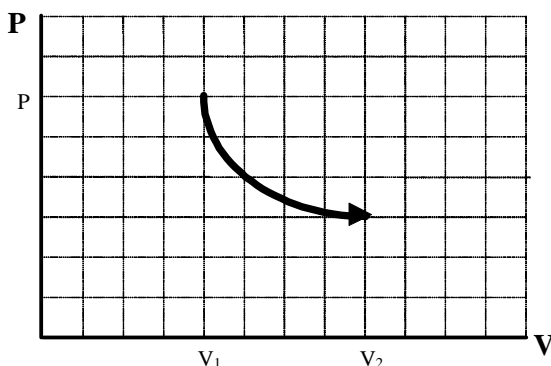
$$W = - p \Delta V$$

$$W = - p (V_2 - V_1)$$

The important realization is that this quantity is exactly equal to the negative of the area under the line representing the process. (The area underneath the line forms a rectangle of width $(V_2 - V_1)$ and height p . Therefore it has an area of $p (V_2 - V_1)$.)

Now, examine a constant temperature process on a pressure vs. volume graph.

Since



$$pV = nRT$$

$$p = \frac{nRT}{V}$$

Thus, a graph of pressure vs. volume should have the distinct shape of an inverse proportion, a *hyperbola*. To determine the work done on a gas during a constant temperature process, we simply need the relationship for the area under a hyperbola. This area, written in terms of thermodynamic variables, is

$$nRT \ln\left(\frac{V_2}{V_1}\right)$$

Thus, the work done on a gas during a constant temperature process is

$$W = -nRT \ln\left(\frac{V_2}{V_1}\right)$$

(The work involves the *natural logarithm* function, ln.)

Back to our specific process,

$$W = -nRT \ln\left(\frac{V_2}{V_1}\right)$$

$$W = -(0.0159)(8.31)(273) \ln\left(\frac{3.57 \times 10^{-4}}{2.0 \times 10^{-4}}\right)$$

$$W = -20.9 \text{ J}$$

Since the temperature is constant, the internal energy is the same in both states. Thus,

$$\frac{3}{2} nRT_1 - 20.9 + Q = \frac{3}{2} nRT_2$$

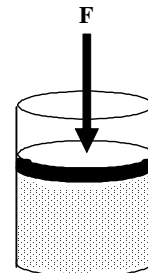
$$\frac{3}{2} nRT_1 - 20.9 + Q = \frac{3}{2} nRT_1$$

$$Q = 20.9 \text{ J}$$

During the removal of the applied force, 20.9 J of energy were transferred to the gas via heat and 20.9 J of energy were transferred out of the gas via the work done by the expanding gas.

An Adiabatic Process

The piston at right is immersed in an ice-water bath. With a 100 N force applied to the circular, 2 cm radius piston head, the volume of gas in the piston is 200 cm³. The external force is quickly removed, leaving only atmospheric pressure pressing on the piston head.



Since the piston is immersed in ice water, the initial temperature of the gas is 0 °C. If the external force is quickly removed, the gas will quickly expand, transferring energy out of the gas via work. In the previous problem, since the expansion was slow, energy was able to flow into the gas, via heat, as this process took place, thus maintaining a constant temperature. If the expansion is fast, there is not sufficient time for energy transfer to take place via heat. A process in which there is no transfer of energy via heat is termed adiabatic. Adiabatic processes occur either very quickly or in systems with thermal insulation.

Event 1: The instant before the force is removed

$$p_1 = 1.81 \times 10^5 \text{ Pa}$$

$$V_1 = 200 \text{ cm}^3 = 2 \times 10^{-4} \text{ m}^3$$

$$T_1 = 273 \text{ K}$$

Event 2: The instant the gas reaches its new equilibrium state

$$p_2 = 1.01 \times 10^5 \text{ Pa}$$

$$V_2 =$$

$$T_2 =$$

Process: Adiabatic

$$W_{12} =$$

$$Q_{12} =$$

The next step is to apply the equation of state to the two states of the gas:

$$p_1 V_1 = nRT_1$$

$$(1.81 \times 10^5 \text{ Pa})(2 \times 10^{-4} \text{ m}^3) = n(8.31 \text{ J/K})(273 \text{ K})$$

$$n = 0.0159 \text{ moles}$$

$$p_2 V_2 = nRT_2$$

$$(1.01 \times 10^5) V_2 = (0.0159)(8.31 \text{ J/K}) T_2$$

Uh-oh, two variables in one equation.

Let's see if the First Law of Thermodynamics can help us out,

$$U_{\text{internal}, i} + W + Q = U_{\text{internal}, f}$$

$$\frac{3}{2} nRT_1 + W + 0 = \frac{3}{2} nRT_2$$

$$W = \frac{3}{2} nR(T_2 - T_1)$$

If we knew the final temperature, we could determine the work done by the gas. If we knew the work done by the gas, we could determine the final temperature. Unfortunately, we don't know either. In previous examples, since one variable was held constant (p, V or T), the equation of state was sufficient to determine the remaining state variables, and the First Law was sufficient to quantify the process. Somehow, from somewhere, we need more information.

Although none of the three thermodynamic variables are constant in an adiabatic process, experiments show that a particular *combination* of thermodynamic variables are constant. This combination (and it is a rather peculiar combination) is the product of the volume raised to the $5/3$ power and the pressure! This exponent is referred to as the *adiabatic constant* and represented by the symbol γ . Thus

$$\gamma = 5/3$$

and

$$p_1 V_1^\gamma = p_2 V_2^\gamma$$

We now have a second relationship between the state variables of a gas, valid only for an adiabatic process. Using this relationship (with $\gamma = 5/3$),

$$\begin{aligned} p_1 V_1^\gamma &= p_2 V_2^\gamma \\ (1.81 \times 10^5)(2.0 \times 10^{-4})^{5/3} &= (1.01 \times 10^5) V_2^{5/3} \\ 0.124 &= (1.01 \times 10^5) V_2^{5/3} \\ 1.23 \times 10^{-6} &= V_2^{5/3} \\ V_2 &= (1.23 \times 10^{-6})^{3/5} \\ V_2 &= 2.84 \times 10^{-4} \text{ m}^3 \end{aligned}$$

Now, by re-examining the second state:

$$\begin{aligned} p_2 V_2 &= nRT_2 \\ (1.01 \times 10^5)(2.84 \times 10^{-4} \text{ m}^3) &= (0.0159)(8.31 \text{ J/K})T_2 \\ T_2 &= 217 \text{ K} \end{aligned}$$

and from the First Law,

$$\begin{aligned} W &= \frac{3}{2} (0.0159)(8.31 \text{ J/K})(217 \text{ K} - 273 \text{ K}) \\ W &= -11.1 \text{ J} \end{aligned}$$

During the abrupt removal of the applied force, 11.1 J of energy were transferred *out* of the gas via the work done by the expanding gas.

Other Processes

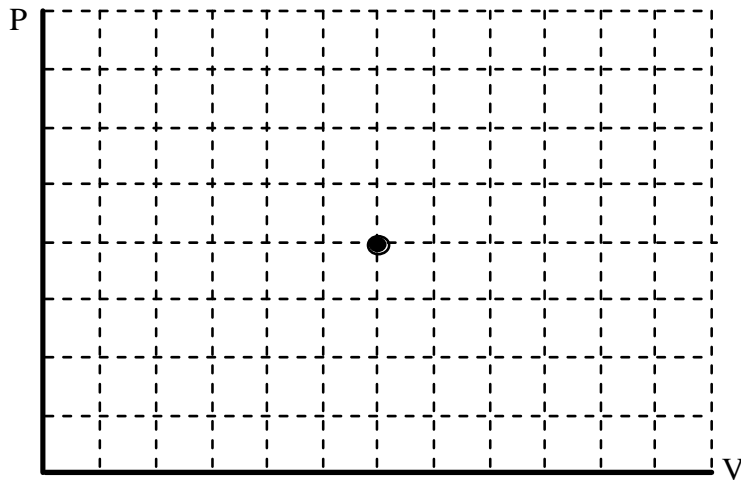
The preceding examples are not meant to imply that all (or even most) thermodynamic processes that occur in nature fall into one of the 4 categories examined above. However, at this level of inquiry, only processes in which one of the thermodynamic state variables is held constant, or the process occurs adiabatically, will be investigated in numerical detail.



The Ideal Gas

Activities

a. Imagine a piston containing a sample of ideal gas. Assume the piston head is perfectly free to move, unless locked in place, and the walls of the piston readily allow the transfer of energy via heat, unless wrapped in insulation. The gas is at the pressure and volume indicated on the graph below and is in equilibrium with a large thermal reservoir at room temperature. Draw a curve, labeled with the appropriate number, to represent each of the following actions. After each action the piston is reset to its initial equilibrium state.

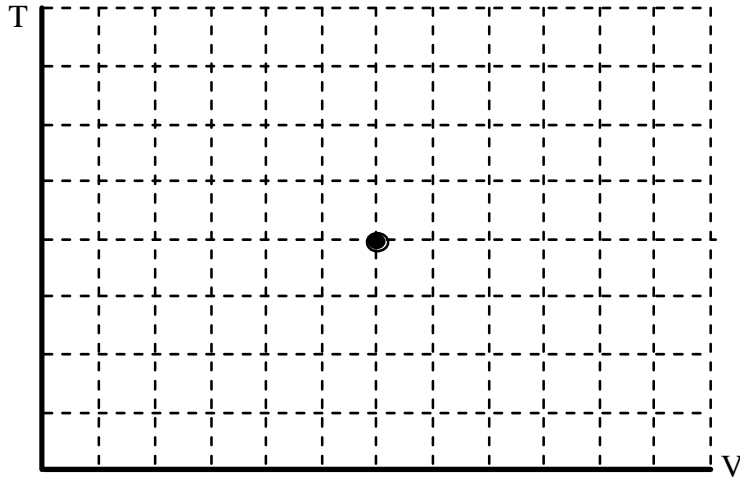


1. Lock the piston head in place. Hold the piston above a very hot flame.
2. Slowly push the piston down.
3. Pull the piston up abruptly.
4. Plunge the piston into very cold water.
5. Wrap the piston in insulation. Slowly pull the piston up.

b. For each of the actions, state whether the energy transferred to the gas via work, the energy transferred to the gas via heat, and the change in the internal energy of the gas are positive, negative, or zero.

Action	W	Q	ΔU
1			
2			
3			
4			
5			

a. Imagine a piston containing a sample of ideal gas. Assume the piston head is perfectly free to move, unless locked in place, and the walls of the piston readily allow the transfer of energy via heat, unless wrapped in insulation. The gas is at the temperature and volume indicated on the graph below and is in equilibrium with a large thermal reservoir at room temperature. Draw a curve, labeled with the appropriate number, to represent each of the following actions. After each action the piston is reset to its initial equilibrium state.

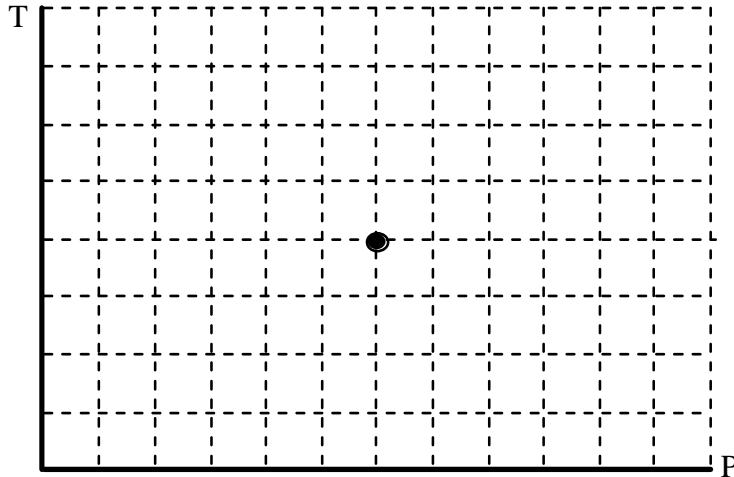


1. Hold the piston above a very hot flame.
2. Push the piston down abruptly.
3. Wrap the piston in insulation. Slowly push the piston down.
4. Lock the piston head in place. Plunge the piston into very cold water.
5. Slowly pull the piston up.

b. For each of the actions, state whether the energy transferred to the gas via work, the energy transferred to the gas via heat, and the change in the internal energy of the gas are positive, negative, or zero.

Action	W	Q	ΔU
1			
2			
3			
4			
5			

a. Imagine a piston containing a sample of ideal gas. Assume the piston head is perfectly free to move, unless locked in place, and the walls of the piston readily allow the transfer of energy via heat, unless wrapped in insulation. The gas is at the temperature and pressure indicated on the graph below and is in equilibrium with a large thermal reservoir at room temperature. Draw a curve, labeled with the appropriate number, to represent each of the following actions. After each action the piston is reset to its initial equilibrium state.

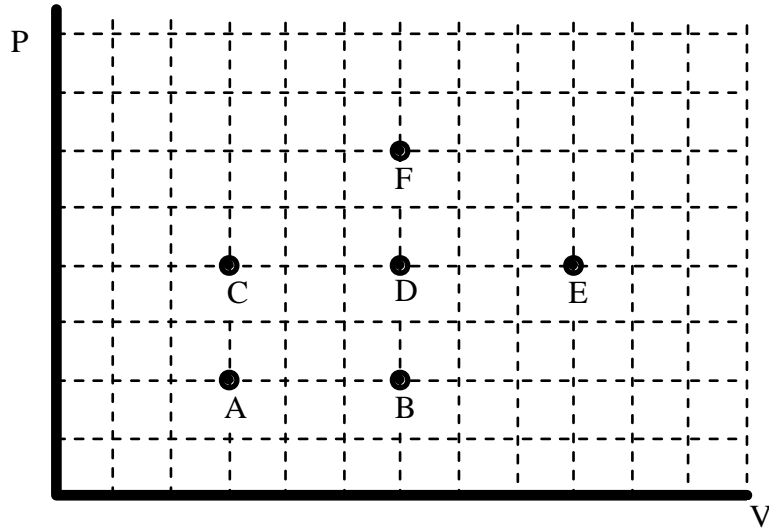


1. Hold the piston above a very hot flame.
2. Push the piston down abruptly.
3. Wrap the piston in insulation. Slowly push the piston down.
4. Lock the piston head in place. Plunge the piston into very cold water.
5. Slowly pull the piston up.

b. For each of the actions, state whether the energy transferred to the gas via work, the energy transferred to the gas via heat, and the change in the internal energy of the gas are positive, negative, or zero.

Action	W	Q	ΔU
1			
2			
3			
4			
5			

Below are representations of six thermodynamic states of the same ideal gas sample.



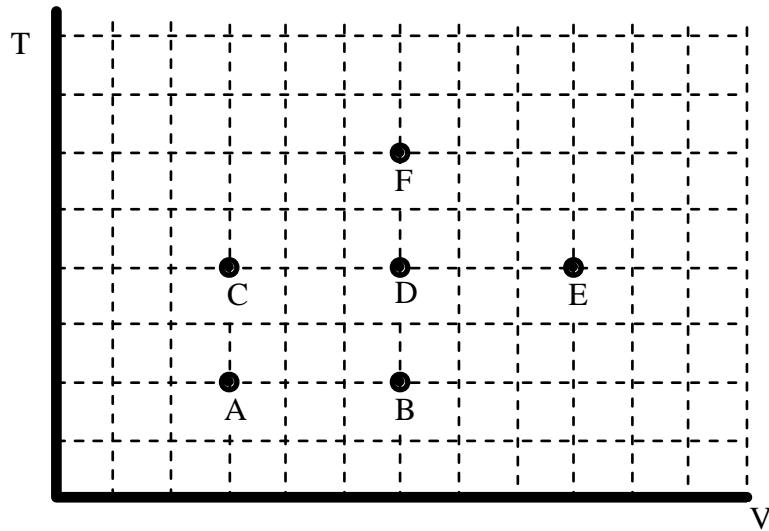
Rank the states on the basis of the temperature of the gas sample at each state.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking can not be determined based on the information provided.

Explain the reason for your ranking:

Below are representations of six thermodynamic states of the same ideal gas sample.



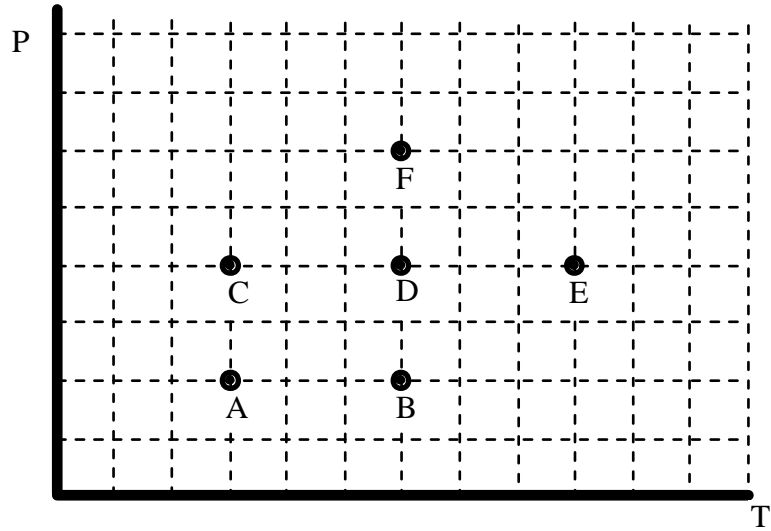
Rank the states on the basis of the pressure of the gas sample at each state.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking can not be determined based on the information provided.

Explain the reason for your ranking:

Below are representations of six thermodynamic states of the same ideal gas sample.



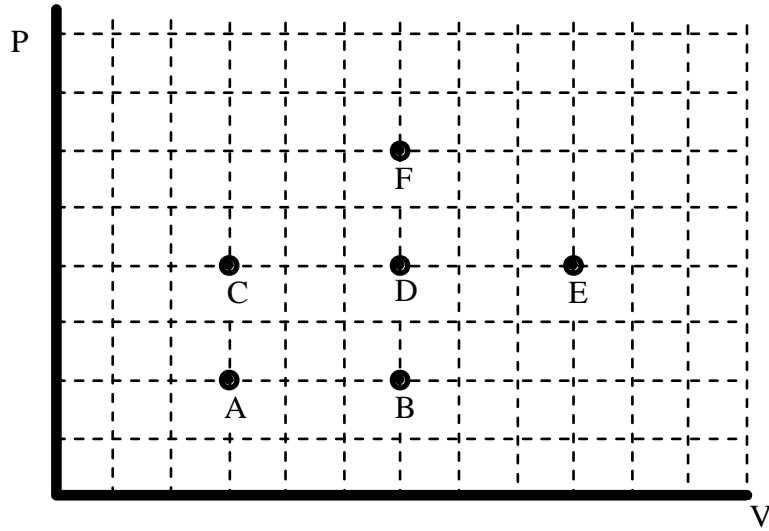
Rank the states on the basis of the volume of the gas sample at each state.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking can not be determined based on the information provided.

Explain the reason for your ranking:

Below are representations of six thermodynamic states of the same ideal gas sample.



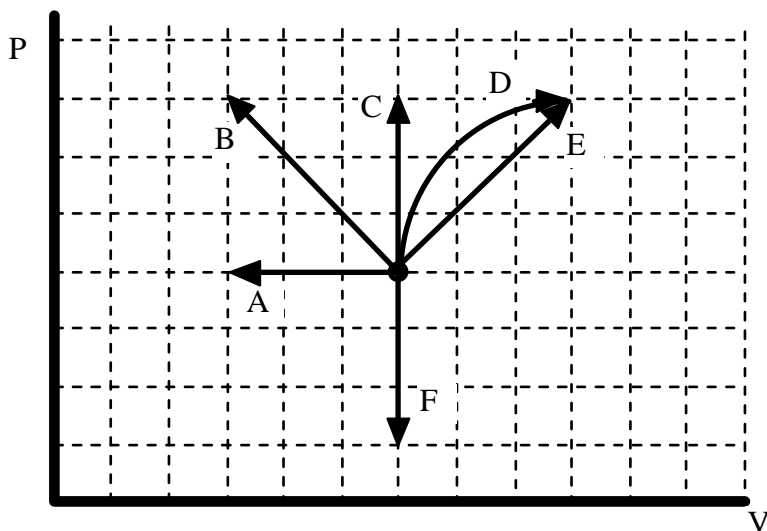
Rank the states on the basis of the average kinetic energy of the atoms in the gas sample at each state.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking can not be determined based on the information provided.

Explain the reason for your ranking:

Below are representations of six thermodynamic processes undertaken with the same ideal gas sample starting from the same initial state. Letters near each end-state label each process.



a. Rank the processes on the basis of the amount of energy transferred to the gas via work during each process. Rank positive work greater than negative work.

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ 6. ____ Smallest

____ The ranking can not be determined based on the information provided.

Explain the reason for your ranking:

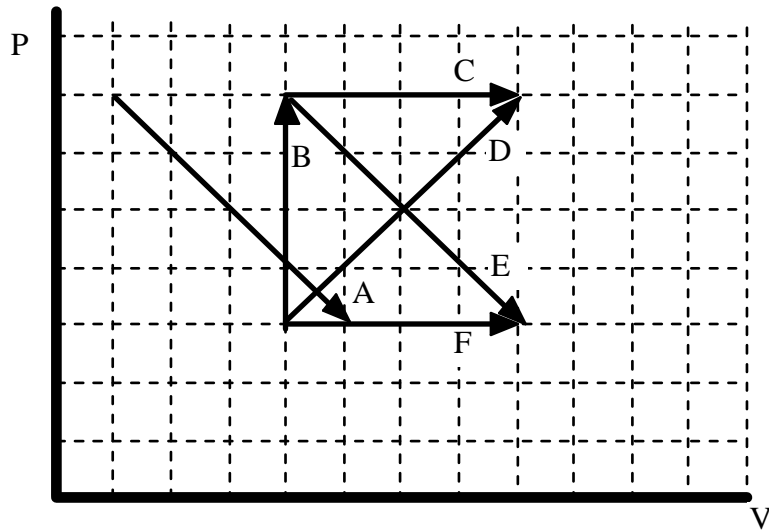
b. Rank the processes on the basis of the change in internal energy of the gas sample. Rank positive changes in internal energy greater than negative changes in internal energy.

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ 6. ____ Smallest

____ The ranking can not be determined based on the information provided.

Explain the reason for your ranking:

Below are representations of six thermodynamic processes undertaken with the same ideal gas sample. Letters near each end-state label each process.



a. Rank the processes on the basis of the amount of energy transferred to the gas via work during each process. Rank positive work greater than negative work.

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ 6. ____ Smallest

____ The ranking can not be determined based on the information provided.

Explain the reason for your ranking:

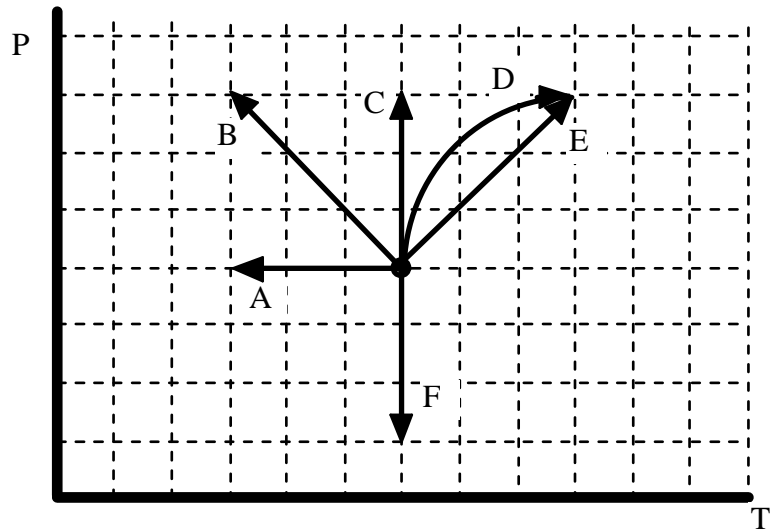
b. Rank the processes on the basis of the change in internal energy of the gas sample. Rank positive changes in internal energy greater than negative changes in internal energy.

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ 6. ____ Smallest

____ The ranking can not be determined based on the information provided.

Explain the reason for your ranking:

Below are representations of six thermodynamic processes undertaken with the same ideal gas sample starting from the same initial state. Letters near each end-state label each process.

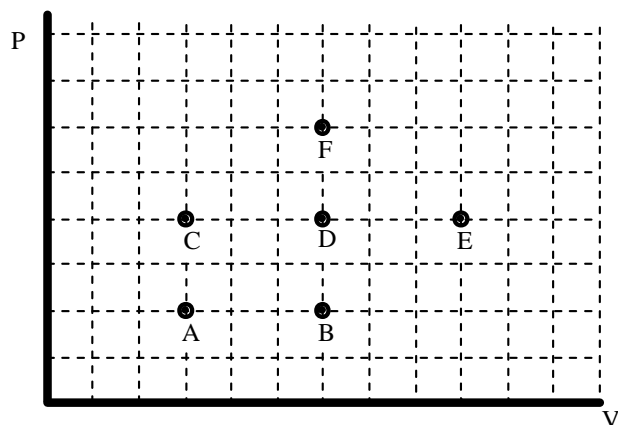


Rank the processes on the basis of the change in internal energy of the gas sample. Rank positive changes in internal energy greater than negative changes in internal energy.

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ 6. ____ Smallest

____ The ranking can not be determined based on the information provided.

Explain the reason for your ranking:



An ideal gas sample is processed between equilibrium states represented by the letters A through F on the pressure vs. volume graph above. All processes connect states via straight line segments.

Compare the energy transferred to the gas via work. Circle the correct comparison symbol.

a. (A to B) > = < ? (B to A)

Explanation:

b. (C to B) > = < ? (B to E)

Explanation:

c. (A to B to D to C to A) > = < ? (C to B to D to F to C)

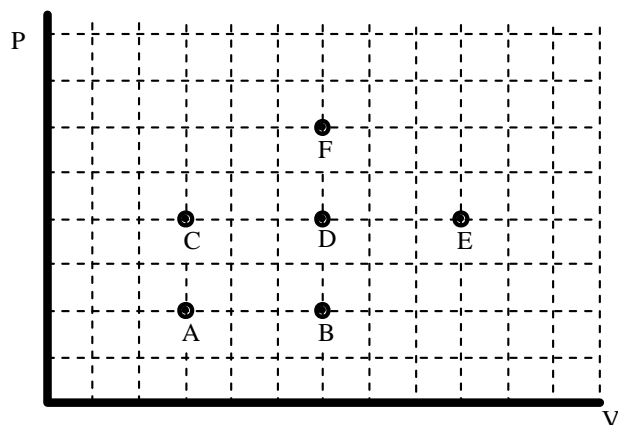
Explanation:

d. (B to A to C to B) > = < ? (B to C to A to B)

Explanation:

e. (E to F to D to E) > = < ? (B to C to D to B)

Explanation:



An ideal gas sample is processed between equilibrium states represented by the letters A through F on the pressure vs. volume graph above. All processes connect states via straight line segments.

Compare the energy transferred to the gas via heat. Circle the correct comparison symbol.

a. **(A to B) > = < ? (B to A)**

Explanation:

b. **(A to B) > = < ? (A to C)**

Explanation:

c. **(A to E) > = < ? (A to F)**

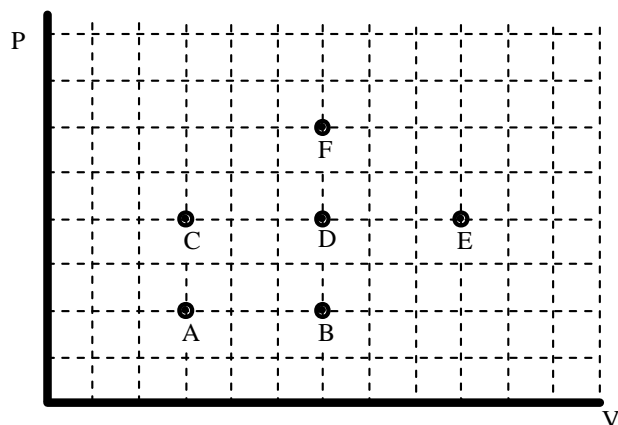
Explanation:

d. **(A to B to D to C to A) > = < ? (C to B to D to F to C)**

Explanation:

e. **(B to A to C to B) > = < ? (B to C to A to B)**

Explanation:



An ideal gas sample is processed between equilibrium states represented by the letters A through F on the pressure vs. volume graph above. All processes connect states via straight line segments.

Compare the change in internal energy of the gas. Circle the correct comparison symbol.

a. **(A to B) > = < ? (B to A)**

Explanation:

b. **(A to D) > = < ? (B to E)**

Explanation:

c. **(A to B to D to C to A) > = < ? (A to B to C to A)**

Explanation:

d. **(B to A to C to B) > = < ? (B to C to A to B)**

Explanation:

e. **(E to F to D to E) > = < ? (B to C to A to B)**

Explanation:

What, if anything, is wrong with the following answers? Correct all the mistakes that you find.

1. Can heat be transferred to an ideal gas sample and the sample's temperature remain constant? If so, describe how this could be done.

No. Temperature is a measure of the amount of heat that has been added to a gas. Therefore, if you transfer additional heat to a gas, the temperature of the gas must increase.

2. Must the pressure always increase when the volume of an ideal gas sample is decreased? Why?

Yes. When a gas sample is compressed, the molecules of the sample now must occupy a smaller volume. Thus, they will collide with the walls of the container more often. More collisions mean a higher pressure.

3. Can work be done on an ideal gas sample and the sample's volume remain constant? If so, describe how this could be done.

Normally, when work is done on a gas sample, the gas volume decreases. However, when heat is normally transferred to a gas sample, the gas volume increases. Therefore, it is possible to both do work on the gas, and transfer heat to the gas, in the proper ratio, such that the volume remains constant.

4. Can the volume of an ideal gas sample decrease even though the pressure and temperature remain constant? If so, describe how this could be done.

No. The equation of state of an ideal gas ($pV = nRT$) clearly indicates that if p and T remain constant, so must V , assuming no gas has leaked out of the sample.

A 5 cubic meter weather balloon is filled with helium at the surface of the earth on a summer (28° C) day. When the balloon reaches an altitude of 6 km, the air pressure has dropped to about half of its original value and the temperature is now - 50° C. Assume the balloon has minimal resistance to expansion.

Thermodynamic Information

Mathematical Analysis

Event 1:

Event 2:

$$P_1 =$$

$$P_2 =$$

$$V_1 =$$

$$V_2 =$$

$$T_1 =$$

$$T_2 =$$

Process:

$$W_{12} =$$

$$Q_{12} =$$

A robotic airplane takes off from the surface of the earth on a summer (26° C) day. When the airplane reaches an altitude of 6 km, the temperature outside has dropped to - 50° C. Assume the airplane was sealed shut at the earth's surface and is made of strong enough material such that its volume does not change.

Thermodynamic Information

Mathematical Analysis

Event 1:

Event 2:

$$P_1 =$$

$$P_2 =$$

$$V_1 =$$

$$V_2 =$$

$$T_1 =$$

$$T_2 =$$

Process:

$$W_{12} =$$

$$Q_{12} =$$

Upon leaving your house on a cold (0°C) morning, you check your tire pressure. The gauge reads 32 lb/in^2 ($2.2 \times 10^5\text{ Pa}$). Upon arriving at school, the gauge reads 35 lb/in^2 . Assume the increase in the volume of the tire is small enough to be ignored. Note that the gauge displays the amount by which the tire pressure exceeds atmospheric pressure.

Thermodynamic Information

Mathematical Analysis

Event 1:

Event 2:

$$P_1 =$$

$$P_2 =$$

$$V_1 =$$

$$V_2 =$$

$$T_1 =$$

$$T_2 =$$

Process:

$$W_{12} =$$

$$Q_{12} =$$

Upon leaving your house on a cold (0°C) morning, you check your tire pressure. The gauge reads 32 lb/in^2 ($2.2 \times 10^5\text{ Pa}$). Upon arriving at school, the gauge reads 35 lb/in^2 . Assume the volume of the tire increased by approximately 5% during the drive. Note that the gauge displays the amount by which the tire pressure exceeds atmospheric pressure.

Thermodynamic Information

Mathematical Analysis

Event 1:

Event 2:

$$P_1 =$$

$$P_2 =$$

$$V_1 =$$

$$V_2 =$$

$$T_1 =$$

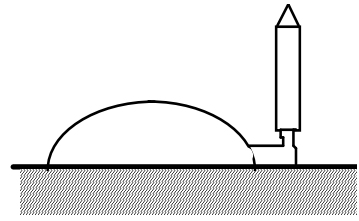
$$T_2 =$$

Process:

$$W_{12} =$$

$$Q_{12} =$$

A stomp rocket is a device in which a sample of air trapped in a heavy-duty, hemispherical balloon is very quickly compressed (by stomping on it) causing an abrupt increase in pressure. This increased pressure launches a toy rocket attached to the gas sample by a nozzle. Assume the gas sample is initially at 20°C and 1.0 atm and the volume of the gas sample is decreased by 50% during the stomp.



Thermodynamic Information

Mathematical Analysis

Event 1:

Event 2:

$p_1 =$

$p_2 =$

$V_1 =$

$V_2 =$

$T_1 =$

$T_2 =$

Process:

$W_{12} =$

$Q_{12} =$

A bubble of volume V is formed at the bottom of the ocean and rises to the top. The pressure at the bottom of the ocean is approximately 6×10^7 Pa and the temperature is 3° C. The temperature of the ocean's surface is approximately 12° C.

Thermodynamic Information

Mathematical Analysis

Event 1:

Event 2:

$$P_1 =$$

$$P_2 =$$

$$V_1 =$$

$$V_2 =$$

$$T_1 =$$

$$T_2 =$$

Process:

$$W_{12} =$$

$$Q_{12} =$$

After diving, a diver has a small bubble of oxygen of volume V trapped in his bloodstream. As his heart beats, the bubble's size varies. His blood pressure is approximately 120 / 80, where '120' is measured in mm of Hg, a unit used to measure pressure. 120 mm of Hg corresponds to a pressure of 1.6×10^4 Pa above atmospheric pressure.

Thermodynamic Information

Mathematical Analysis

Event 1:

Event 2:

$$P_1 =$$

$$P_2 =$$

$$V_1 =$$

$$V_2 =$$

$$T_1 =$$

$$T_2 =$$

Process:

$$W_{12} =$$

$$Q_{12} =$$

The pressure in a fluid as a function of depth beneath the surface (for fluids at a uniform temperature) is $p = p_o + \rho g d$, where p_o is the pressure at the surface, ρ is the density of the fluid, and d is the distance below the surface. A bubble of volume V is formed at the bottom of a 0.9 m high 'yard of beer' and rises to the top. The density of beer is approximately $1.0 \times 10^3 \text{ kg/m}^3$.

Thermodynamic Information

Mathematical Analysis

Event 1:

Event 2:

$$p_1 =$$

$$p_2 =$$

$$V_1 =$$

$$V_2 =$$

$$T_1 =$$

$$T_2 =$$

Process:

$$W_{12} =$$

$$Q_{12} =$$

In an automobile engine, an air-fuel mixture initially at atmospheric pressure and 85° C is quickly compressed by a factor of four before ignition. Assume the mixture behaves like an ideal gas.

Thermodynamic Information

Mathematical Analysis

Event 1:

Event 2:

$$P_1 =$$

$$P_2 =$$

$$V_1 =$$

$$V_2 =$$

$$T_1 =$$

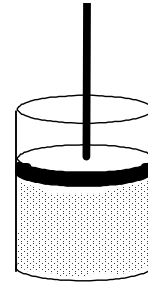
$$T_2 =$$

Process:

$$W_{12} =$$

$$Q_{12} =$$

An amateur inventor claims that the following process will “remove heat” from the surroundings of the piston, a water bath at 80° C. Initially, the volume of the gas in the piston is 200 cm³ and the pressure exerted on the piston head is 1.5 atm above atmospheric pressure. The external pressure is quickly removed, leaving only atmospheric pressure on the piston head. Once the gas stops its rapid expansion, the piston head is locked in place and the sample is allowed to return to thermal equilibrium with the water bath.

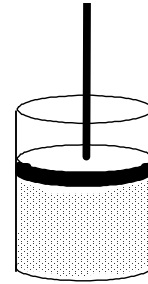


Thermodynamic Information

Event 1:	Event 2:	Event 3:	Event 4:	Event 5:
$P_1 =$	$P_2 =$	$P_3 =$	$P_4 =$	$P_5 =$
$V_1 =$	$V_2 =$	$V_3 =$	$V_4 =$	$V_5 =$
$T_1 =$	$T_2 =$	$T_3 =$	$T_4 =$	$T_5 =$
Process:	Process:	Process:	Process:	
$W_{12} =$	$W_{23} =$	$W_{34} =$	$W_{45} =$	
$Q_{12} =$	$Q_{23} =$	$Q_{34} =$	$Q_{45} =$	

Mathematical Analysis

An amateur inventor claims that the following process will “remove heat” from the surroundings of the piston, a water bath at 80°C . Initially, the volume of the gas in the piston is 200 cm^3 and the pressure exerted on the piston head is 1.5 atm above atmospheric pressure. The external pressure is quickly removed, leaving only atmospheric pressure on the piston head. Once the gas stops its rapid expansion, the piston head is locked in place and the sample is allowed to return to thermal equilibrium with the water bath. Then, the external pressure is slowly increased until the sample returns to its original volume.

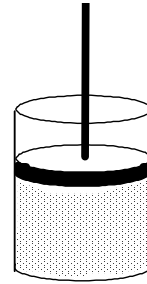


Thermodynamic Information

Event 1:	Event 2:	Event 3:	Event 4:	Event 5:
$P_1 =$	$P_2 =$	$P_3 =$	$P_4 =$	$P_5 =$
$V_1 =$	$V_2 =$	$V_3 =$	$V_4 =$	$V_5 =$
$T_1 =$	$T_2 =$	$T_3 =$	$T_4 =$	$T_5 =$
Process:	Process:	Process:	Process:	
$W_{12} =$	$W_{23} =$	$W_{34} =$	$W_{45} =$	
$Q_{12} =$	$Q_{23} =$	$Q_{34} =$	$Q_{45} =$	

Mathematical Analysis

An amateur inventor is unsure what effect the following process will have on the surroundings of the piston, a water bath at 80°C . Initially, the volume of the gas in the piston is 200 cm^3 and the pressure exerted on the piston head is 1.5 atm above atmospheric pressure. The external pressure is very slowly removed, until only atmospheric pressure remains on the piston head. Once the gas stops its expansion, an additional external pressure is quickly applied to the piston head until the sample returns to its original volume. Then, the piston head is locked in place and the sample is allowed to return to thermal equilibrium with the water bath.

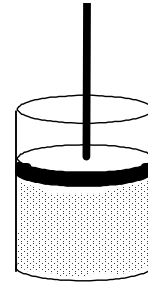


Thermodynamic Information

Event 1:	Event 2:	Event 3:	Event 4:	Event 5:
$P_1 =$	$P_2 =$	$P_3 =$	$P_4 =$	$P_5 =$
$V_1 =$	$V_2 =$	$V_3 =$	$V_4 =$	$V_5 =$
$T_1 =$	$T_2 =$	$T_3 =$	$T_4 =$	$T_5 =$
Process:	Process:	Process:	Process:	
$W_{12} =$	$W_{23} =$	$W_{34} =$	$W_{45} =$	
$Q_{12} =$	$Q_{23} =$	$Q_{34} =$	$Q_{45} =$	

Mathematical Analysis

An amateur inventor has no idea what he's doing. He starts with 400 cm^3 of gas in the piston at atmospheric pressure, with the piston in an ice water bath. After locking the piston head in place, he then plunges the piston into a hot water bath (90° C). Once thermal equilibrium is reached, he unlocks the piston head and allows the gas sample to slowly expand. Once the expansion is complete, he returns the sample to the ice water bath (without locking the piston head).

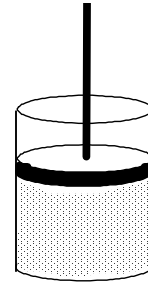


Thermodynamic Information

Event 1:	Event 2:	Event 3:	Event 4:	Event 5:
$P_1 =$	$P_2 =$	$P_3 =$	$P_4 =$	$P_5 =$
$V_1 =$	$V_2 =$	$V_3 =$	$V_4 =$	$V_5 =$
$T_1 =$	$T_2 =$	$T_3 =$	$T_4 =$	$T_5 =$
Process:	Process:	Process:	Process:	
$W_{12} =$	$W_{23} =$	$W_{34} =$	$W_{45} =$	
$Q_{12} =$	$Q_{23} =$	$Q_{34} =$	$Q_{45} =$	

Mathematical Analysis

An amateur inventor has no idea what she's doing. She starts with 400 cm^3 of gas in the piston at atmospheric pressure, with the piston in an ice water bath. Attaching a vacuum pump, she slowly reduces the pressure on the piston head to 0.3 atm . After locking the piston head in place, she removes the vacuum pump. She then places the piston in a flame until the pressure of the gas is once again 1.0 atm . Once this is complete, she unlocks the piston head and she returns the sample to the ice water bath.

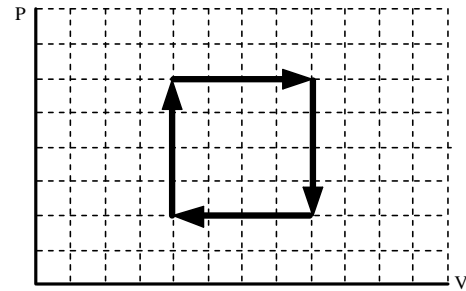


Thermodynamic Information

Event 1:	Event 2:	Event 3:	Event 4:	Event 5:
$P_1 =$	$P_2 =$	$P_3 =$	$P_4 =$	$P_5 =$
$V_1 =$	$V_2 =$	$V_3 =$	$V_4 =$	$V_5 =$
$T_1 =$	$T_2 =$	$T_3 =$	$T_4 =$	$T_5 =$
Process:	Process:	Process:	Process:	
$W_{12} =$	$W_{23} =$	$W_{34} =$	$W_{45} =$	
$Q_{12} =$	$Q_{23} =$	$Q_{34} =$	$Q_{45} =$	

Mathematical Analysis

In a certain industrial process, thirty moles of gas are processed through the cycle at right. The gas begins the cycle at a pressure of 1.5 atm and temperature 300 K. The first step of the cycle is an expansion to twice the initial volume. The minimum temperature of the gas during the cycle is 180 K. The graph is not to scale.

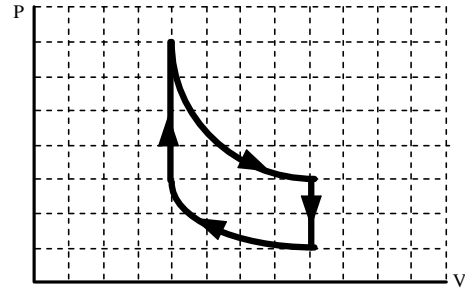


Thermodynamic Information

Event 1:	Event 2:	Event 3:	Event 4:	Event 5:
$P_1 =$	$P_2 =$	$P_3 =$	$P_4 =$	$P_5 =$
$V_1 =$	$V_2 =$	$V_3 =$	$V_4 =$	$V_5 =$
$T_1 =$	$T_2 =$	$T_3 =$	$T_4 =$	$T_5 =$
Process:	Process:	Process:	Process:	
$W_{12} =$	$W_{23} =$	$W_{34} =$	$W_{45} =$	
$Q_{12} =$	$Q_{23} =$	$Q_{34} =$	$Q_{45} =$	

Mathematical Analysis

In a certain industrial process, a gas sample is processed through the cycle at right. The gas starts at a pressure of 2 atm, volume of 0.6 m^3 and a temperature of 450 K. The gas expands isothermally (at constant temperature) to twice its original volume. The pressure is then decreased at constant volume. The gas is then isothermally compressed to its original volume and a pressure of 1.7 atm. The gas is then returned to its initial state. The graph is not to scale.



Thermodynamic Information

Event 1:	Event 2:	Event 3:	Event 4:	Event 5:
$P_1 =$	$P_2 =$	$P_3 =$	$P_4 =$	$P_5 =$
$V_1 =$	$V_2 =$	$V_3 =$	$V_4 =$	$V_5 =$
$T_1 =$	$T_2 =$	$T_3 =$	$T_4 =$	$T_5 =$
Process:	Process:	Process:	Process:	
$W_{12} =$	$W_{23} =$	$W_{34} =$	$W_{45} =$	
$Q_{12} =$	$Q_{23} =$	$Q_{34} =$	$Q_{45} =$	

Mathematical Analysis