Electromagnetic Waves

Concepts and Principles

Putting it All Together

Electric charges are surrounded by electric fields. When these charges move, the electric field changes. Additionally, these moving charges (or, more importantly, these changing electric fields) create magnetic fields. Since before the motion no magnetic field existed, this is a change in the value of the magnetic field (from zero to non-zero). By electromagnetic induction, this change in magnetic field creates emf, which is a change in the value of the electric field in a region of space. But this change in electric field must cause further change in the magnetic field, which must cause further change in the electric field, which must …

Sorting through this interrelationship between electric and magnetic fields, which involves the simultaneously solution of a set of coupled, partial-differential equations, is the grand accomplish of James Maxwell. This work of Maxwell’s, in the 1860s, is generally regarded as on par with the work of Isaac Newton and Albert Einstein. Although we won’t actually solve this set of equations, we will study several of the consequences of Maxwell’s work.

Electromagnetic Waves

By far the most important conclusion from Maxwell’s work is that the changes in electric and magnetic fields, coupled together as described above, propagate through space as an electromagnetic wave. Near the moving charge that created the wave, the mathematical description of the wave is very complicated, but once the wave has moved a “reasonable” distance from the charge that created it, it can be visualized as below:

![Electromagnetic Wave Diagram](image)
and mathematically represented as:

\[ E = E_{\text{max}} \cos\left(\frac{2\pi x}{\lambda} - 2\pi ft\right) \]
\[ B = B_{\text{max}} \cos\left(\frac{2\pi x}{\lambda} - 2\pi ft\right) \]

where

- \( \lambda \) is the wavelength of the wave, the distance between maximum values of the field vectors,
- \( f \) is the frequency of the wave, the number of complete cycles of oscillation that wave passes through per second,
- the electric and magnetic field vectors are perfectly in phase, meaning they pass through their maximum and minimum values at the same points in space and time,
- the field vectors are oriented at 90° from each other,
- and the wave propagates in the +x-direction. The direction of propagation can be determined by pointing the fingers of your right hand in the direction of the electric field vector and then curling your fingers until they point in the direction of the magnetic field. The direction your thumb points is the direction of travel of an electromagnetic wave.

Of course, a similar mathematical description could be given for an electromagnetic wave propagating in any direction.

It’s important to realize that the “picture” of the wave above is for a specific instant of time. This is a traveling wave, in that it moves through space, in this case in the +x-direction. Since it moves through space, an important characteristic of the wave is its speed. While solving the set of equations, Maxwell found that the speed of electromagnetic waves, denoted \( c \), is given by the expression:

\[ c = \frac{1}{\sqrt{\mu_0/k}} \]

where \( k \) and \( \mu_0 \) are the familiar electrostatic constant and permeability of free space.

What is somewhat odd about this result is that it states that the speed of the wave is a universal constant, since both \( \varepsilon_0 \) and \( \mu_0 \) are constant. This means that regardless of the wavelength of the wave, or its frequency, or whether the charges that created the wave were wiggling in a stationary radio antenna on top of a mountain or in the headlight of a spaceship moving at a billion miles per hour (or if you viewed the wave from a second spaceship moving a billion mph in the other direction!) the wave always moves at exactly the same speed. The wave speed is completely independent of how it was created. Once the wave is created, its motion through space is completely determined by how its electric and magnetic fields interact with each other, and its speed has no relationship to the physical charge that created it. (This fact will have some curious repercussions about 40 years after Maxwell’s discovery.)
Plugging in the known values for k and \(\mu_0\) yields:

\[
c = 3.0 \times 10^8 \text{ m/s}
\]

Maxwell realized that this was the known speed for visible light, and soon came to the conclusion that light was, in fact, an electromagnetic wave. Electromagnetic waves of other frequencies, such as radio waves, microwaves, x rays, etc, all propagate at this same speed and obey the same mathematical framework described above.

One final interesting conclusion from Maxwell’s solution was that the ratio of the electric field to the magnetic field at any point in the wave has a constant value, equal to the wave speed:

\[
\frac{E}{B} = c
\]

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**Energy of Electromagnetic Waves**

Energy can be stored in electric and magnetic fields, as you learned when you analyzed capacitors and inductors in electric circuits. Since electromagnetic waves involve the propagation of changes in these fields, they should involve the flow of energy in the direction of the wave’s motion.

The *intensity*, or energy flow per unit area, of an electromagnetic wave is given by:

\[
S = \frac{E^2}{\mu_0 c} \quad \text{or} \quad S = \frac{c B^2}{\mu_0}
\]

(These two expressions are equivalent because \(\frac{E}{B} = c\).)

This expression gives you the intensity at any point in the electromagnetic wave at one specific instant of time. However, in many ways this isn’t particularly useful. If you are dealing with a visible light wave, for example, the frequency of the wave is approximately \(10^{14}\) cycles per second. This means that the intensity of the wave cycles through maximum and minimum values every \(10^{14}\) s!
A much more useful expression would be for the average intensity of the wave. To find this expression, substitute our description of the electric field-portion of an electromagnetic wave

\[ E = E_{\text{max}} \cos\left(\frac{2\pi x}{\lambda} - 2\pi f t\right) \]

into the above expression

\[ S = \frac{E^2}{c\mu_0} \]
\[ S = \frac{(E_{\text{max}} \cos(\frac{2\pi x}{\lambda} - 2\pi f t))^2}{c\mu_0} \]
\[ S = \frac{E_{\text{max}}^2 \cos^2(\frac{2\pi x}{\lambda} - 2\pi f t)}{c\mu_0} \]

To find the average value of this function we need only to find the average value of the cosine-squared function, since all of the other terms are constant. It’s common knowledge\(^1\) that the average value of the cosine-squared function is \( \frac{1}{2} \). Therefore, the average intensity of an electromagnetic wave is:

\[ \overline{S} = \frac{E_{\text{max}}^2}{2c\mu_0} \]

or equivalently

\[ \overline{S} = \frac{cB_{\text{max}}^2}{2\mu_0} \]

---

\(^1\) It is now.
Momentum of Electromagnetic Waves

Just as electromagnetic waves involve the flow of energy, they also carry momentum in the direction of motion. Although it may seem confusing that mass-less, non-material fields, or at least changes in these fields, can have momentum, the momentum carried by electromagnetic waves can be directly measured. The long tail of a comet, pointing directly away from the sun, is a result of the momentum carried by the sun’s electromagnetic waves.

The simplest phenomenon directly involving the momentum of electromagnetic waves occurs when the waves are absorbed or scattered from an object. Since the wave has momentum, some of this momentum is transferred to the object the wave “collides” with. This interaction can be directly studied by applying conservation of momentum, but an easier shortcut involves defining the concept of \textit{radiation pressure}. Radiation pressure is the force per unit area that the wave exerts on the object during the collision. Radiation pressure is defined as:

\[ p_{\text{rad}} = \frac{\vec{S}}{c} \]

where \( \alpha \) is a number between 1 and 2 that depends on the portion of the electromagnetic wave absorbed during the collision. If the wave if fully absorbed, \( \alpha = 1 \). If the wave is perfectly reflected, \( \alpha = 2 \). (We won’t analyze situations between these two extremes.)

Polarization of Electromagnetic Waves

In many cases, the electric field component of an electromagnetic wave oscillates in a well-defined direction. When this is the case, the wave is said to be \textit{polarized}. The polarization direction is the direction in which the electric field oscillates. For example, in the diagram below the wave is polarized in the y-direction.
Certain materials allow only one specific direction of electric field vector to propagate through them. These materials, called polarizers, typically absorb (or reflect) all waves with electric field vectors not aligned with their transmission axis. For example, a polarizer with transmission axis along the +x-direction would completely block the propagation of the wave illustrated above.

If the incident electric field vector is not perfectly aligned with the transmission axis, only the component of the field vector along the transmission axis can pass through the polarizer. Thus the electric field magnitude that passes through the polarizer, \( E \), is given by:

\[
E = E_0 \cos \theta
\]

where

- \( E_0 \) is the electric field magnitude incident on the polarizer,
- \( \theta \) is the angle between the incident polarization direction and the transmission axis.

After passing through the polarizer, the electromagnetic wave is now polarized along the transmission axis.

Rather than concentrating on the electric field magnitude, in most cases its more useful to focus on the intensity of the wave. Since the average intensity of the wave is given by

\[
\bar{S} = \frac{E_{\text{max}}^2}{2c\mu_0}
\]

the intensity is proportional to the square of the electric field vector. Thus, a more useful version of the polarization equation is:

\[
S = S_0 \cos^2(\theta)
\]

where

- \( S_0 \) is the intensity incident on the polarizer,
- \( \theta \) is the angle between the incident polarization direction and the transmission axis.
Interference of Electromagnetic Waves

Since electromagnetic waves consist of alternating electric and magnetic field vectors, if two or more waves pass through the same point in space (at the same time) their field vectors must add according to the basic principles of vector addition. This combination of waves through the vector addition of their field vectors is called interference.

In general, the interference of two electromagnetic waves can be incredibly difficult to analyze. We will restrict ourselves to a specific sub-class of interference phenomenon, in which there are only two sources of waves and the two sources produce waves of exactly the same frequency perfectly in phase. Typically, the easiest way to accomplish this is by having a single source of waves and then somehow dividing the waves and sending each portion of the wave along a separate path to a final common location. (This is a lot simpler than it sounds.)

The interference that results when the two waves are recombined depends only on the path length difference, $\Delta d$, the waves traveled to reach the recombination location. The path length difference is, as the name implies, simply the difference between the distance traveled by wave #1 and the distance traveled by wave #2 between separation and recombination,

$$\Delta d = |d_{traveled\ by\ wave\ #2} - d_{traveled\ by\ wave\ #1}|$$

If this difference is exactly equal to zero, one wavelength, or any integer number of wavelengths, the waves will be perfectly in phase when they recombine and exhibit constructive interference, resulting in a locally maximum value for the electric field vector. Therefore, for constructive interference:

$$\Delta d = m\lambda$$

where $m$ is zero or any integer.

If the path length difference is exactly equal to one-half a wavelength, or any half-integer number of wavelengths, the waves will be perfectly out of phase when they recombine and exhibit destructive interference, resulting in a locally minimum value for the electric field vector. Therefore, for destructive interference:

$$\Delta d = (m + \frac{1}{2})\lambda$$

where $m$ is zero or any integer.
Electromagnetic Waves

Analysis Tools

Energy, Power and Intensity

Future lunar colonists may want to watch their favorite earthly TV shows. Suppose a television station on Earth has a power of 1.0 MW and broadcasts isotropically.

a. What is the intensity of this signal on the moon?

b. What total power would be received by a 20 cm radius “satellite” dish on the moon?

c. What is the maximum electric field strength of this signal on the moon?

If the signal is broadcast isotropically (the same in all directions), its intensity should decrease as the surface area of the sphere over which the signal spreads increases. Since the distance to the moon is 3.82 x 10^8 m,

\[ S = \frac{\text{power}}{\text{area}} \]

\[ S_{\text{at moon}} = \frac{1.0 \times 10^6}{4\pi(3.82 \times 10^8)^2} \]

\[ S_{\text{at moon}} = 5.5 \times 10^{-13} \text{ W/m}^2 \]

The power received by the dish would be

\[ S = \frac{\text{power}}{\text{area}} \]

\[ \text{Power} = S_{\text{at moon}} A_{\text{of dish}} \]

\[ \text{Power} = (5.5 \times 10^{-13} \text{ W/m}^2)(\pi(0.20 \text{ m})^2) \]

\[ \text{Power} = 6.9 \times 10^{-14} \text{ W} \]
This corresponds to a maximum electric field of

\[ \overline{S} = \frac{E_{\text{max}}^2}{2c\mu_0} \]

\[ E_{\text{max}} = \sqrt{2\overline{S}\mu_0c} \]

\[ E_{\text{max}} = 2.0 \times 10^{-5} \text{ N/C} \]

**Pressure**

*High-power lasers are used to compress a small hydrogen pellet in an attempt to initiate fusion. A laser generating pulses of radiation of power 1.5 GW is focused isotropically onto the 0.5 mm radius pellet. What is the pressure exerted on the pellet if the pellet absorbs the incident light?*

When an electromagnetic wave is absorbed by a surface, it exerts a pressure on the surface given by

\[ p_{\text{rad}} = (1) \frac{\overline{S}}{c} \]

The intensity of the light on the surface of the pellet is

\[ S = \frac{\text{power}}{\text{area}} \]

\[ S = \frac{1.5 \times 10^9}{4\pi(0.0005)^2} \]

\[ S = 4.77 \times 10^{14} \text{ W/m}^2 \]

Thus, the pressure on the pellet is

\[ p = \frac{S}{c} \]

\[ p = \frac{4.77 \times 10^{14}}{3 \times 10^8} \]

\[ p = 1.59 \times 10^6 \text{ N/m}^2 \]
Polarization

Initially unpolarized light is sent through three polarizing sheets with transmission axes oriented at $\theta_1=0^\circ$, $\theta_2=45^\circ$, and $\theta_3=90^\circ$ measured counterclockwise from the x-axis. What percentage of the initial intensity is transmitted by the system of the three sheets?

If polarized light of intensity $S_0$ is incident on a polarizing sheet, the intensity transmitted by the sheet depends on the angle between the polarization direction of the light and the transmission axis of the sheet, $\theta$, via

$$S = S_0 \cos^2(\theta)$$

But what if the incident light is unpolarized?

If the incident wave is unpolarized, we can imagine that it is a combination of waves of all possible polarizations. Then, to determine the intensity that passes through the polarizer we can average over all of these different hypothetical polarization directions. Since the intensity that passes through the polarizer depends on the square of the cosine function, this means we must find the average value of the cosine-squared function. Since it’s common knowledge\(^2\) that the average value of the cosine-squared function is $\frac{1}{2}$, the intensity after the first sheet is one-half of the initial intensity:

$$S_{after1} = \frac{1}{2} S_0$$

and the wave is now polarized along the x-axis.

After the second sheet,

$$S_2 = S_1 \cos^2(\theta)$$

$$S_2 = \left(\frac{1}{2} S_0 \right) \cos^2(45^\circ)$$

$$S_2 = 0.25 S_0$$

and the wave is now polarized at $45^\circ$ from the x-axis.

\(^2\) It should be by now.
After the third sheet,

\[ S_3 = S_2 \cos^2(\theta) \]
\[ S_3 = (0.25S_0) \cos^2(45) \]
\[ S_3 = 0.125S_0 \]

and the wave is now polarized along the y-axis. (Notice that \( \theta \) is not the angle of the transmission axis (90°), but rather the angle between the previous polarization direction and the transmission axis.) Thus, 12.5% of the initial light intensity passes through the three sheets and the resulting light is vertically polarized.

### Two Source Interference

Two radar sources are separated by \( d = 10 \text{ m} \). During testing, the two sources broadcast a test frequency perfectly in phase. If destructive interference occurs at \( x = 40 \text{ m} \), what are the possible values for the source wavelength?

Destructive interference occurs when the two waves have a relative phase difference of \( \pi \) (one wave is “flipped over” relative to the other). Since the two waves are emitted in phase, this phase difference must be due to the different distances the waves travel to reach the point of destructive interference. For their phase difference to equal \( \pi \), their path length difference must be a half-integer multiple of their wavelength.

With the top source labeled #1 and the bottom source #2:

\[ \Delta d = |d_{\text{traveled by wave #2}} - d_{\text{traveled by wave #1}}| \]
\[ \Delta d = \sqrt{d^2 + x^2} - x \]
\[ \Delta d = \sqrt{10^2 + 40^2} - 40 \]
\[ \Delta d = 1.23 \text{ m} \]
For destructive interference,

\[ \Delta d = (m + \frac{1}{2})\lambda \]

\[
\begin{align*}
m = 0: & \quad m = 1: & \quad m = 2: \\
1.23 = \frac{1}{2}\lambda & \quad 1.23 = \frac{3}{2}\lambda & \quad 1.23 = \frac{5}{2}\lambda \\
\lambda = 2.46m & \quad \lambda = 0.82m & \quad \lambda = 0.49m
\end{align*}
\]

The source could have any of the above (or many other) wavelengths.

**Double Slit Interference**

Monochromatic green light of wavelength 550 nm illuminates two parallel narrow slits \( d = 8.00 \mu m \) apart. If the interference pattern is projected on a screen \( D = 5.0 \) m from the slits, where are the bright fringes (constructive maxima) located on the screen?

Constructive interference occurs when the two waves have a relative phase difference of 0°. Since the two waves are emitted in phase, any phase difference must be due to the different distances the waves travel to reach the various points on the screen. For their phase difference to be 0°, their path length difference must be an integer multiple of their wavelength.

We could proceed exactly as in the previous example and directly calculate the path length difference between the two waves that recombine on the screen. However, since the distance to the screen (D) is much, much larger than the separation between the slits (d) we can make use of a simplifying observation.

Let’s imagine we are interested in the location on the screen indicated below.
Although the paths from each slit toward the screen are not precisely parallel, if the drawing was to scale they would be extremely close to parallel. Below is a blow-up of the region near the slits:

```
<p>| |</p>
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
</tr>
</tbody>
</table>
```

By adding a line perpendicular to the two parallel paths, and noting that this line makes the same angle $\theta$ with the vertical that the paths make with the horizontal, the path length difference between these two paths is just the small distance indicated in the diagram below:

```
<p>| |</p>
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
</tr>
</tbody>
</table>
```

so

$$\Delta d = d \sin \theta$$

where

$$\tan \theta = \frac{y}{D}$$

from the diagram on the previous page. Although these results are only approximate, when $D \gg d$ they are very useful.

Since we are looking for constructive interference in this example,

$$\Delta d = m\lambda$$

$$d \sin \theta = m\lambda$$

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right)$$

$$\theta = \sin^{-1}\left(\frac{m(550 \times 10^{-9})}{8.00 \times 10^{-5}}\right)$$

$$\theta = 0^\circ, 3.94^\circ, 7.90^\circ, \text{ etc.}$$
These angles correspond to

\[
\tan \theta = \frac{y}{D} \\
y = D \tan \theta \\
y = (5.0) \tan \theta \\
y = 0\text{m}, 0.344\text{m}, 0.694\text{m}, \text{etc.}
\]

### Thin Film Interference

A camera lens with index of refraction 1.40 is coated with a thin transparent film of index of refraction 1.20 to eliminate the reflection of blue light (\(\lambda = 480\text{ nm}\)) normal to its surface. What is the minimum thickness of the film?

If blue light is not reflected from this coated camera lens, this means that waves that reflect from the front of the lens coating (the air-coating interface) and waves that reflect from the rear of the lens coating (the coating-lens interface) destructively interfere for \(\lambda = 480\text{ nm}\) light. These waves may have a difference in phase for two, independent reasons.

First, when a wave front reflects from a material having a larger index of refraction than the medium it is currently traveling through, it will reflect with a complete phase inversion (i.e., the wave will “flip over” when reflecting from something with a larger index).

Second, the wave reflecting from the rear of the coating will have traveled a greater distance than the wave reflecting from the front of the coating. This difference in path length will also cause a phase shift between the two waves.

For this particular example, both the front-reflecting wave and the rear-reflecting wave reflect from surfaces having a larger index than they currently are traveling through, so both waves are “flipped” upon reflection. Since both waves are flipped, this effect will not influence the relative phase of the two waves.

Calling the thickness of the coating \(d\), the rear-reflecting wave travels a distance \(2d\) further than the front-reflecting wave. Thus,

\[\Delta d = 2d\]

If this distance is equal to one-half (or three-halves, etc.) wavelengths, the reflected interference will be destructive.

\[2d = (m + \frac{1}{2})\lambda\]
However, there is one last complication. The wavelength of the light in the coating is not equal to the wavelength of the light in air, and $2d$ must equal one-half of the wavelength of the light in the coating for destructive reflection. The wavelength of light in a material other than vacuum is given by:

$$\lambda_{\text{material}} = \frac{\lambda_{\text{vacuum}}}{n_{\text{material}}}$$

Combining this with the previous relationship yields

$$2d = (m + \frac{1}{2})(\frac{\lambda}{n})$$

$$2d = (0 + \frac{1}{2})(\frac{480\text{nm}}{1.20})$$

$$d = 100\text{nm}$$

Thus, the coating must have a minimum thickness of 100 nm.
Electromagnetic Waves

Activities
A neodymium-glass laser can provide 100 TW of power in 1.0 ns pulses at a wavelength of 0.26 μm.

a. How much energy is contained in a single pulse?

**Mathematical Analysis**

b. What is the minimum beam diameter such that if the laser is fired through air it does not cause electrical breakdown? (Air electrically breaks down at electric fields of 3 x 10⁶ N/C.)

**Mathematical Analysis**
A helium-neon laser has a beam power of 5.00 mW at a wavelength of 633 nm. The beam is focused to a circular spot whose effective diameter is equal to 2.00 wavelengths.

a. What is the maximum electric field magnitude in the focused beam?

**Mathematical Analysis**

b. What is the maximum magnetic field magnitude in the focused beam?

**Mathematical Analysis**
Our closest stellar neighbor, Proxima Centauri, is 4.3 light-years away. It has been suggested that TV programs from our planet have reached this star and may have been viewed by the hypothetical inhabitants of its solar system. Suppose a television station on earth has a power of 1.0 MW, and radiates isotropically.

a. What is the intensity of this signal at Proxima Centauri?

**Mathematical Analysis**

b. What electric field magnitude would the Centaurians have to be able to detect to watch our old TV shows?

**Mathematical Analysis**
The radiation emitted by a laser spreads out in the form of a narrow cone with circular cross section. The angle \( \theta \) of the cone is called the full-angle beam divergence. An argon laser, radiating 300 kW at 514.5 nm, is aimed at the moon in a ranging experiment.

a. If the beam has a full angle beam divergence of 0.880 \( \mu \text{rad} \), what area on the moon’s surface does the laser illuminate?

Mathematical Analysis

b. What is the intensity of the beam on the moon’s surface?

Mathematical Analysis
Frank D. Drake, an investigator in the SETI (Search for Extra-Terrestrial Intelligence) program, once said that the large radio telescope in Arecibo, Puerto Rico, could detect a signal that strikes the earth with a total power of only one picowatt.

a. What power would be received by the Arecibo antenna for such a signal? (The antenna diameter is 300 m.)

Mathematical Analysis

b. What power source would be required at the center of our galaxy to provide such a signal? (The galactic center is $2.2 \times 10^5$ lightyears away.)

Mathematical Analysis
Radar works by bouncing electromagnetic waves off of targets and detecting the reflected wave. Imagine an airplane flying at a distance of 50 km from a 35 kW radar transmitter. Assume the transmitter broadcasts over a half-sphere, all the power incident on the plane is reflected into a half-sphere, and the airplane has an effective area of 50 m².

a. What is the power reflected from the plane?

**Mathematical Analysis**

b. What is the intensity of the reflected signal detected at the transmitter?

**Mathematical Analysis**
High-power lasers are used to compress a small hydrogen pellet in an attempt to initiate fusion. A laser generating pulses of radiation of power 1.5 GW is focused isotropically onto the 1.0 mm$^2$ surface area pellet.

a. What is the pressure exerted on the pellet if the pellet absorbs the incident light?

Mathematical Analysis

b. What is the maximum value of the electric field at the surface of the pellet?

Mathematical Analysis
Radiation from the sun reaching the earth has an intensity of 1.4 kW/m².

a. What is the total power radiated by the sun? (The sun is $1.5 \times 10^{11}$ m from earth.)

**Mathematical Analysis**

b. Assuming all the incident energy is absorbed, what is the force on the earth due to radiation pressure?

**Mathematical Analysis**
You are 1.0 m from a 150 W light bulb.

a. What is the maximum electric and magnetic field on your skin due to the bulb?

**Mathematical Analysis**

b. What is the force on each square inch of your body due to radiation pressure? Compare this value to normal air pressure.

**Mathematical Analysis**
A laser beam of power 6.0 W and diameter 2.0 mm is directed upward at a highly reflective oil droplet of density 0.85 g/cm³. What maximum radius droplet can be levitated by the radiation pressure of the laser beam?

Mathematical Analysis
A small spaceship with mass 2000 kg (including an astronaut) is drifting in outer space. The astronaut uses a 10 MW laser beam as a propulsion device, firing it out the rear of the space craft.

a. What is the acceleration of the spaceship?

**Mathematical Analysis**

b. How long will it take the ship to reach a speed of 25,000 m/s?

**Mathematical Analysis**
It has been proposed that a spaceship might be propelled in the solar system by radiation pressure, using a large sail made of foil. How large must the sail be if the radiation force is equal in magnitude to the sun's gravitational attraction? Assume that the mass of the ship and sail is 300 kg and that the sail is perfectly reflecting. (The power output of the sun is $3.9 \times 10^{26}$ W and the mass of the sun is $1.99 \times 10^{30}$ kg.)

Mathematical Analysis
All particles in the solar system are under the combined influence of the sun's gravitational attraction and the radiation pressure from the sun's electromagnetic waves. Imagine a highly reflective spherical particle of density $1.0 \times 10^3$ kg/m$^3$. Below what radius will the particle be blown out of the solar system?

Mathematical Analysis
Initially unpolarized light is sent through three polarizing sheets with transmission axes oriented at $\theta_1 = 30^\circ$, $\theta_2 = 60^\circ$, and $\theta_3 = 90^\circ$ measured counterclockwise from the x-axis.

a. What is the polarization direction of the exiting wave?

b. What percentage of the initial intensity is transmitted by the system of the three sheets?

**Mathematical Analysis**
Vertically polarized light is sent through three polarizing sheets with transmission axes oriented at $\theta_1 = 30^\circ$, $\theta_2 = 60^\circ$, and $\theta_3 = 90^\circ$ measured counterclockwise from the $x$-axis.

a. What is the polarization direction of the exiting wave?

b. What percentage of the initial intensity is transmitted by the system of the three sheets?

**Mathematical Analysis**
A beam of partially polarized light (a mixture of vertically polarized and unpolarized light) of intensity 500 µW/mm² is incident on a polarizing sheet. In order to determine the relative intensities of the polarized and unpolarized portions of the beam, the light is sent through a vertical polarizer, and the intensity is noted to drop to 300 µW/mm². The polarizer is then rotated until horizontal, and for this orientation the transmitted intensity is 200 µW/mm². Determine the intensities of the polarized and unpolarized portions of the beam.

Mathematical Analysis
A beam of partially polarized light (a mixture of horizontally polarized and unpolarized light) is incident on a polarizing sheet. In order to determine the relative intensities of the polarized and unpolarized portions of the beam, the light is sent through a vertical polarizer, and the intensity is noted to drop by 80%. The polarizer is then rotated until horizontal, and for this orientation the intensity drops by 20%. Determine the relative intensities of the polarized and unpolarized portions of the beam.

**Mathematical Analysis**
Sending a beam of polarized light through a series of polarizing sheets can rotate the direction of polarization. If you want to rotate the beam’s polarization by 90° and only lose 15% of the intensity of the beam, what is the minimum number of sheets necessary, assuming each sheet is rotated by the same amount relative to the initial polarization direction?

Mathematical Analysis
Sending a beam of polarized light through a series of polarizing sheets can rotate the direction of polarization. You want to rotate the polarization of a 500 µW/mm² vertically polarized beam by 90°. However, your polarizing sheets will melt if they absorb an intensity of 100 µW/mm². Describe the minimum system of polarizing sheets necessary to rotate the polarization of the beam.

**Mathematical Analysis**
Monochromatic green light of wavelength 550 nm illuminates two parallel narrow slits 7.70 \( \mu \text{m} \) apart. If the interference pattern is projected on a screen 5.0 m from the slits, what is the distance between adjacent bright fringes?

Mathematical Analysis
Monochromatic light illuminates two parallel narrow slits 600 µm apart. When the interference pattern is projected on a screen 3.0 m from the slits, the distance between adjacent bright fringes is 3.0 mm. What is the wavelength of the light?

Mathematical Analysis
Microwaves are coherently broadcast from a pair of speaker cones $d = 6.0$ cm apart. An intensity maximum is detected at $D = 75$ cm and $\theta = 12^\circ$. What are the possible values for the microwave's wavelength?

Mathematical Analysis
3.0 cm wavelength microwaves are coherently broadcast from a pair of speaker cones. An intensity maximum is detected at $D = 2.0 \text{ m}$ and $\theta = 8^\circ$. What is the separation between the speakers?

Mathematical Analysis
Two coherent radio sources broadcast at $\lambda = 2.5 \text{ m}$ and are separated by $d = 10 \text{ m}$. At what values of $x$ does destructive interference occur?

Mathematical Analysis
Two coherent radio sources are separated by \( d = 8 \text{ m} \). If constructive interference occurs at \( x = 25 \text{ m} \), what are the possible values for the source wavelength?

**Mathematical Analysis**
Light of wavelength 624 nm is incident normal to a soap film (n = 1.33) suspended in air.

a. What thicknesses of film will fully reflect the light?

Mathematical Analysis

b. Assuming the film has its minimum thickness, is the film reflective for any other visible wavelengths?

Mathematical Analysis
A camera lens with index of refraction 1.47 is coated with a thin transparent film of index of refraction 1.25 to eliminate the reflection of blue light (\( \lambda = 475 \text{ nm} \)) normal to its surface.

a. What is the minimum thickness of the film?

**Mathematical Analysis**

b. Is the film non-reflective for any other visible wavelengths?

**Mathematical Analysis**
The rhinestones in costume jewelry are glass with index of refraction 1.50, coated with a layer of silicon monoxide with index 2.00.

a. What is the minimum coating thickness needed to ensure that normal incident light of wavelength 550 nm is constructively reflected? Are any visible wavelengths not seen on reflection?

Mathematical Analysis

b. Are any visible wavelengths not seen on reflection?

Mathematical Analysis
A disabled tanker leaks kerosene \((n = 1.20)\) into the Persian Gulf, creating a large slick on top of the water \((n = 1.30)\).

a. If you are looking straight down from an airplane at a region of the slick where its thickness is 460 nm, which wavelength(s) of visible light do you see reflected?

**Mathematical Analysis**

b. If you are scuba diving directly under this same region, which wavelength(s) of visible light do you see transmitted?

**Mathematical Analysis**
Two glass plates and a wire form the system shown at right. The plates are 120 mm long, touch at the left end, and are separated by a wire of unknown diameter at the right end. A broad beam of $\lambda = 680$ nm light is sent directly downward through the top plate. The $17^{th}$ dark fringe of the reflection interference pattern occurs directly above the midpoint of the plate. What is the wire’s diameter?

Mathematical Analysis
Two glass plates and a wire form the system shown at right. The plates are 60 mm long, touch at the left end, and are separated by a wire of diameter 0.08 mm at the right end. A broad beam of $\lambda = 400$ nm light is sent directly downward through the top plate.

a. Is the left edge of the transmission interference pattern light or dark?

b. What order is the transmission interference pattern fringe that occurs directly below the midpoint of the lower plate?

Mathematical Analysis
At right is a lens of diameter 4.5 cm and radius of curvature 380 cm lying on a glass plate and illuminated from above by light with wavelength 550 nm. If viewed from above the lens, circular interference fringes appear, associated with the variable thickness $d$ of the air film between the lens and the plate. How many bright circular fringes appear for this configuration?

**Mathematical Analysis**
At right is a lens of radius 1.6 cm and unknown radius of curvature lying on a glass plate and illuminated from above by light with wavelength 550 nm. If viewed from above the lens, circular interference fringes appear, associated with the variable thickness $d$ of the air film between the lens and the plate. The outer edge of the pattern corresponds to the $755^{th}$ dark fringe. What is the radius of curvature of the lens?

Mathematical Analysis