Electric Circuits

*Concepts and Principles*

**Electric Circuits as Applied Physics**

Electric circuits are one of the most practical applications of our understanding of electric and magnetic fields. In general, an electric circuit is any device that consists of a closed path for charges to move (a current), a source of energy to “drive” the motion of the charges (a potential difference, or voltage, often in the form of a battery), and various circuit elements that can either convert (resistors) or store (capacitors and inductors) the energy supplied by the energy source.

The study of circuits is incredibly broad, since there are limitless ways to combine these elements into an electric circuit. We will restrict ourselves to studying circuits with only a limited number of elements, and with a source that supplies a constant voltage\(^1\).

---

**The Resistor**

In general, a *resistor* is any device that converts electrical energy into another form of energy, often heat. For example, a fluorescent light bulb converts electrical energy into light (with about a 20% efficiency, the remaining energy is converted into heat) and an incandescent light bulb converts electrical energy very efficiently into heat (with only about 5% of the incident energy converted to light). Since a conversion of electrical energy takes place in these devices, they are resistors.

In all resistors, the electric potential energy of the charges entering the device is larger than the electric potential energy of the charges exiting the device, because some of the potential energy has been converted to other forms. This decrease in potential energy is due to a decrease in electric potential between the two ends of the device and is directly proportional to the *resistance* of the device.

---

\(^1\) Circuits with constant voltage sources are referred to as DC, or direct current, circuits.
The definition of resistance for a device is:

\[ R = \frac{\Delta V}{i} \]

where

- \( \Delta V \) is the potential difference between the two ends of the device, often termed the \textit{voltage drop} across the device,
- and \( i \) is the current that flows through the device.

The unit of resistance, \( \frac{V}{A} \), is defined as the \textit{ohm} (Ω).

The previous expression relates the resistance of a resistor to properties of the circuit it is part of. However, it is also sometimes useful to directly relate the resistance to the actual physical parameters of the device itself. For simple, passive resistors (basically blocks of material connected to a voltage source), resistance is defined as:

\[ R = \frac{\rho L}{A} \]

where

- \( \rho \) is the \textit{resistivity} of the material from which the resistor is constructed,
- \( L \) is the length of the resistor in the direction of current flow,
- and \( A \) is the cross-sectional area of the resistor.

Resistivity can range from 0 for a perfect conductor to \( \infty \) for a perfect insulator.

One final note on the properties of resistors concerns their rate of energy conversion. Since electric potential is the electric potential energy per unit of charge, and current is the charge flowing through the device per second, the product of change in electric potential and current is the change in electric potential energy per second. Thus, the rate of energy conversion, or \textit{power}, in a resistor is given by:

\[ P = i(\Delta V) \]
The Capacitor

A capacitor is a device that stores energy in the electric field between two closely spaced conducting surfaces. When connected to a voltage source, electric charge accumulates on the two surfaces but, since the conducting surfaces are separated by an insulator, the charges cannot travel from one surface to the other. The charges create an electric field in the space between the surfaces, and the two surfaces have a difference in electric potential.

\[ \Delta V \]

Once “charged”, if the capacitor is removed from the original circuit and connected to a second circuit it can act as a voltage source and “drive” its collected charge through the second circuit. When used in this way, the capacitor clearly acts as a temporary storehouse of energy.

To determine the energy stored in a capacitor, we first need to define the capacitance of the capacitor. The capacitance of a capacitor is defined as:

\[ C = \frac{Q}{\Delta V} \]

where

- \( Q \) is the magnitude of the electric charge stored on either conducting surface,
- \( \Delta V \) is the potential difference between the surfaces.

The unit of capacitance, \( \frac{C}{V} \), is defined as the farad (F).

The amount of energy that can be stored on a capacitor is a function of both its capacitance and the potential difference between its surfaces. The relationship between stored energy and these parameters is:

\[ U = \frac{1}{2} C(\Delta V)^2 \]
**The Inductor**

An *inductor* is a device that stores energy in the magnetic field created when current passes through a coil of wire. When connected to a voltage source, current will flow though the inductor, establishing a magnetic field.

If the voltage source is suddenly removed, current will continue to flow in the coil because of electromagnetic induction. This induced current will act to replace the disappearing source current. The energy needed to drive this current comes from the energy stored in the magnetic field, so in this case the inductor acts as a temporary storehouse of energy.

To determine the energy stored in an inductor, we first need to define the *inductance* of the inductor. The inductance of the inductor is defined as:

\[
L = \frac{\Phi}{i}
\]

where

- \(\Phi\) is the magnetic flux within the inductor,
- and \(i\) is the current flowing through the inductor.

The unit of inductance, \(\frac{Tm^2}{A}\), is defined as the *henry* (H).

The amount of energy that can be stored in an inductor is a function of both its inductance and the current flowing through it. The relationship between stored energy and these parameters is:

\[
U = \frac{1}{2} Li^2
\]
Electric Circuits

Analysis Tools

Resistors in Circuits

The circuit at right represents a 12 V car battery and two mismatched headlights, $R_1 = 1.9 \, \Omega$ and $R_2 = 2.1 \, \Omega$.

a. Determine the magnitude of the potential difference across and the current through each circuit component.

b. If the battery has a total stored energy of 800 W hr, and produces a constant potential difference until discharged, how long will the bulbs stay lit?

The potential difference across the car battery is given as 12 V. This means that the electric potential in the wire coming out of the “top” of the battery is 12 V larger than the potential in the wire coming from the “bottom”. Since each of the resistors are attached to these same two wires, the top of each resistor is 12 V higher in potential than the bottom. Therefore the potential difference across each resistor is 12 V. When circuit elements are connected such that the elements all have the same potential difference, the elements are said to be in parallel.

Since the potential difference across each resistor is known, we can use the definition of resistance to calculate the current through each branch of the circuit. Analyzing branch #1 yields

\[
R_1 = \frac{\Delta V}{i_1} \\
i_1 = \frac{\Delta V}{R_1} \\
i_1 = \frac{12V}{1.9\Omega} \\
i_1 = 6.32 A
\]
and branch #2

\[ R_2 = \frac{\Delta V}{i_2} \]
\[ i_2 = \frac{\Delta V}{R_2} \]
\[ i_2 = \frac{12V}{2.1\Omega} \]
\[ i_2 = 5.71A \]

The current that flows through R_1 and the current that flows through R_2 must also flow through both the top and bottom wires connected to the battery. To complete the mental image of a closed circuit of current, we will say the current flows “through” the battery as well, although this is not technically true. Therefore, the current that flows through the battery (the total current flowing in the circuit) is:

\[ i_{\text{battery}} = i_1 + i_2 \]
\[ i_{\text{battery}} = 6.32A + 5.71A \]
\[ i_{\text{battery}} = 12.0A \]

We can summarize this information in a simple table:

<table>
<thead>
<tr>
<th></th>
<th>( \Delta V_{\text{across}} ) (V)</th>
<th>( i_{\text{through}} ) (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>battery</td>
<td>12</td>
<td>12.0</td>
</tr>
<tr>
<td>R_1</td>
<td>12</td>
<td>6.32</td>
</tr>
<tr>
<td>R_2</td>
<td>12</td>
<td>5.71</td>
</tr>
</tbody>
</table>

To determine how long the headlights will stay lit, we must calculate the total power of the circuit (the total amount of electrical energy converted per second). We can do this separately for each headlight and then add the results:

\[ P_1 = i_1(\Delta V_1) \]
\[ P_1 = (6.32A)(12V) \]
\[ P_1 = 75.8W \]

and
\[ P_2 = i_2(\Delta V_2) \]
\[ P_2 = (5.71A)(12V) \]
\[ P_2 = 68.5W \]

so the total power of the circuit is:

\[ P_{\text{total}} = 144.3W \]

Therefore, the battery will last for

\[
\frac{800W \text{ hrs}}{144.3W} = 5.54\text{hrs}
\]

**Capacitor Properties**

*Imagine a pair of long, hollow nested cylinders of inner radius a and outer radius b. Calculate the capacitance, per meter, for these nested cylinders.*

Since capacitance is defined by the relation

\[ C = \frac{Q}{\Delta V} \]

we need to determine the potential difference that would develop between these cylinders if charges Q (and –Q) were placed on the two surfaces.

To do this, imagine that a charge +Q (per meter) was placed on the inner cylinder. Using Gauss’ Law, this leads to an electric field between the cylinders of:

\[ E = \frac{Q}{2\pi\varepsilon_0 r} \]

This field is directed radially away from the central axis of the cylinders.
Once the electric field between the cylinders is known, the magnitude of the potential difference between the cylinders can be calculated by:

\[ \Delta V = -\int \vec{E} \cdot d\vec{l} \]

\[ \Delta V = \int_{a}^{b} \frac{Q}{2\pi\varepsilon_{0}r} \hat{r} \cdot d\hat{r} \]

\[ \Delta V = \int_{a}^{b} \frac{Q}{2\pi\varepsilon_{0}r} \, dr \]

\[ \Delta V = \frac{Q}{2\pi\varepsilon_{0}} \frac{b}{a} \ln(b/a) \]

Substituting this result into the definition of capacitance yields:

\[ C = \frac{Q}{\Delta V} \]

\[ C = \frac{Q}{2\pi\varepsilon_{0}} \frac{b}{a} \ln(b/a) \]

\[ C = \frac{2\pi\varepsilon_{0}}{\ln(b/a)} \]

Thus, the capacitance per meter of a set of nested cylinders depends on the natural logarithm of the ratio of the cylinder radii. Notice that if the cylinders are very close together (b is not much larger than a), the capacitance is very large. The capacitance of a capacitor is always enhanced by having the two charged surfaces very close together. However, as the surfaces get closer together, the possibility of electrical breakdown (charges “jumping” across the gap) becomes larger. For this reason, and several others, the space between the surfaces in a capacitor is typically filled with a type of material, called a dielectric, which both enhances the capacitance of the capacitor and inhibits electrical breakdown.
The device at right represents a simplified camera flash circuit. With \( V = 3 \, \text{V} \) and \( R = 100 \, \Omega \), find \( C \) such that the flash reaches 80% of its final voltage in 1.0 s.

The circuit above, termed an \( RC \) circuit, can best be analyzed by considering the changes in electric potential experienced by a hypothetical charge “journeying” around the circuit:

- as it “passes through” the battery the potential increases by \( V \),
- as it passes through the resistor the potential decreases by

\[
R = \frac{\Delta V_R}{i} \\
\Delta V_R = iR
\]

- and as it “passes through” the capacitor the potential decreases by

\[
C = \frac{\Delta V_C}{Q} \\
\Delta V_C = \frac{Q}{C}
\]

Putting these changes in potential together results in:

\[
V - \Delta V_R - \Delta V_C = 0 \\
V - iR - \frac{Q}{C} = 0
\]

Note that the total change in potential (and potential energy) must be zero since the energy given to the charge by the battery is partially converted by the resistor and partially stored by the capacitor.
If we take a time derivative of the above equation (noting that $V$, $R$, and $C$ are constants, but that $Q$, the charge on the capacitor, is changing) we are left with a differential equation for the current in the circuit:

\[
\frac{d}{dt}(V - iR - \frac{Q}{C}) = \frac{d}{dt}(0)
\]

\[
0 - R \frac{di}{dt} - \frac{1}{C} \frac{dQ}{dt} = 0
\]

\[
-R \frac{di}{dt} - \frac{1}{C} i = 0
\]

\[
\frac{di}{dt} = -\frac{1}{RC} i
\]

This equation says that the time derivative of the current is equal to the product of the current and the numerical factor $-\frac{1}{RC}$. The only mathematical function that has the property that its derivative is proportional to itself is the exponential function. Therefore, the current must be given by the function:

\[
i(t) = i_0 e^{-t/RC}
\]

where $i_0$ is the current at $t = 0$ s.

If we assume that the capacitor is uncharged when the switch is first closed, then

\[
V - iR - \frac{Q}{C} = 0
\]

\[
V - i_0 R - \frac{(0)}{C} = 0
\]

\[
i_0 = \frac{V}{R}
\]

so the final expression for the current in the circuit as a function of time is:

\[
i(t) = \frac{V}{R} e^{-t/RC}
\]

Using this expression we can determine the time-dependence of any other circuit parameter.
For example, the question asks about the voltage across the capacitor. Since the voltage across the resistor can be expressed as:

\[ R = \frac{\Delta V_R}{i} \]

\[ \Delta V_R(t) = Ri \]

\[ \Delta V_R(t) = R\left(\frac{V}{R}e^{-t/RC}\right) \]

\[ \Delta V_R(t) = Ve^{-t/RC} \]

the voltage across the capacitor is the amount of the source voltage that “remains”:

\[ V - \Delta V_R - \Delta V_C = 0 \]

\[ \Delta V_C = V - \Delta V_R \]

\[ \Delta V_C = V - Ve^{-t/RC} \]

\[ \Delta V_C = V\left(1 - e^{-t/RC}\right) \]

This function shows that after a long time \((t \to \infty)\), the voltage across the capacitor will equal the voltage of the source.

Therefore,

\[ \Delta V_C = V\left(1 - e^{-t/RC}\right) \]

\[ 0.8(3) = (3)(1 - e^{-1/100C}) \]

\[ 0.8 = 1 - e^{-1/100C} \]

\[ e^{-1/100C} = 0.2 \]

\[ -\frac{1}{100C} = \ln(0.2) \]

\[ C = 6.21 \text{ mF} \]

Thus, a 6.21 mF capacitor will reach 80% of its final voltage in 1.0 s.
**Inductor Properties**

Imagine a pair of long, hollow nested wires of inner radius \(a\) and outer radius \(b\), designed to carry current into and out of the page. Calculate the inductance, per meter, for these nested wires.

Since inductance is defined by the relation

\[
L = \frac{\Phi}{i}
\]

we need to determine the flux that would develop between these wires if current \(i\) (and \(-i\)) flowed along the two wires.

To do this, imagine that current \(i\) flowed out of the page along the inner wire. Using Ampere’s Law, this leads to a magnetic field between the cylinders of:

\[
B = \frac{\mu_0i}{2\pi r}
\]

To help calculate the flux between the wires, the diagram at right is a top view of the nested wires. The dashed area is the area over which we will calculate the flux. (The current along the inner wire flows toward the top of the page, resulting in magnetic field pointing directly out of the page in the area of interest.)

The shaded sliver is the differential element, located a distance \(r\) from the center of the wires, with thickness \(dr\) and length \(l\). The magnetic flux is then:

\[
\Phi = \int B \cdot dA
\]

\[
\Phi = \int_a^b \left(\frac{\mu_0i}{2\pi r}\right)(ldr)
\]

\[
\Phi = \frac{\mu_0il}{2\pi} \int_a^b \frac{dr}{r}
\]

\[
\Phi = \frac{\mu_0i}{2\pi} \ln\left(\frac{b}{a}\right)
\]
Substituting this result into the definition of inductance yields:

\[ L = \frac{\Phi}{i} \]
\[ L = \frac{\mu_0 il \ln\left(\frac{b}{a}\right)}{2\pi} \]
\[ L = \frac{\mu_0 l}{2\pi} \ln\left(\frac{b}{a}\right) \]

The inductance per meter is then:

\[ L = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right) \]

Thus, the inductance per meter of a set of nested wires depends on the natural logarithm of the ratio of the wire radii. Notice that if the wires are very far apart, the inductance is larger. However, as the wires get farther apart, the size of the device gets larger and may become impractical. For this reason, and several others, the space containing the magnetic flux in an inductor is typically filled with a material with a high magnetic permeability, like iron, in order to “concentrate” the magnetic flux into a smaller region of space.
The device at right represents a simplified electromagnet. With $V = 100$ V and $R = 15 \, \Omega$, find $L$ such that the current reaches 5.0 A in 0.5 s.

The circuit above, termed an RL circuit, can best be analyzed by considering the changes in electric potential experienced by a hypothetical charge “journeying” around the circuit:

- as it “passes through” the battery the potential increases by $V$,
- as it passes through the resistor the potential decreases by
  \[ R = \frac{\Delta V_R}{i} \]
  \[ \Delta V_R = iR \]
- and as it “passes through” the inductor the potential changes by
  \[ L = \frac{\Phi}{i} \]
  \[ \Phi = Li \]

Since by Faraday’s Law of Induction,

\[ \varepsilon = -\frac{d\Phi}{dt} \]
\[ \varepsilon = -\frac{d(Li)}{dt} \]
\[ \varepsilon = -L \frac{di}{dt} \]

The emf induced by the inductor is the potential drop across it, so

\[ \Delta V_L = -L \frac{di}{dt} \]
Putting these changes in potential together results in:

\[ V - iR - L \frac{di}{dt} = 0 \]

Again, note that the total change in potential (and potential energy) must be zero since the energy given to the charge by the battery is partially converted by the resistor and partially stored by the inductor.

If we take a time derivative of the above equation (noting that V, R, and L are constants) we are left with a differential equation for the current in the circuit:

\[ \frac{d}{dt} \left( V - iR - L \frac{di}{dt} \right) = \frac{d}{dt} (0) \]

\[ 0 - R \frac{di}{dt} - L \frac{d^2i}{dt^2} = 0 \]

\[ \frac{d^2i}{dt^2} = -\frac{R}{L} \frac{di}{dt} \]

This equation says that the time derivative of the derivative of the current is equal to the product of the derivative of the current and a numerical factor. This means that the derivative of the current must be an exponential function. Therefore, the derivative of the current must be given by the function:

\[ \frac{di}{dt} = Ae^{-Rt/L} \]

where A is an arbitrary constant. Integrating this result leads to a current of the form:

\[ i(t) = Be^{-Rt/L} + D \]

where B and D are arbitrary constants.

To determine these constants, consider the current in the circuit after a very long time \( t \to \infty \). After this amount of time the circuit will have reached an equilibrium value, so the change in the current will be zero. Thus,

\[ V - iR - L \frac{di}{dt} = 0 \]

\[ V - i_\infty R - L(0) = 0 \]

\[ i_\infty = \frac{V}{R} \]
Therefore,

\[ i(t) = Be^{-Rt/L} + D \]
\[ i(\infty) = Be^{-R(\infty)/L} + D \]
\[ \frac{V}{R} = 0 + D \]
\[ D = \frac{V}{R} \]

Now consider the current in the circuit the instant you first close the switch \((t \to 0)\). At this instant, no current can be flowing in the circuit. This is because if there was current flowing instantaneously after the switch was closed, this would be a discontinuous change in current and the inductor would create an infinite emf to oppose this “infinite” increase in current. Therefore,

\[ i(t) = Be^{-Rt/L} + \frac{V}{R} \]
\[ i(0) = Be^{-R(0)/L} + \frac{V}{R} \]
\[ 0 = B + \frac{V}{R} \]
\[ B = -\frac{V}{R} \]

Now that we know the values of the two constants, the final expression for the current in the circuit as a function of time is:

\[ i(t) = \frac{V}{R} (1 - e^{-Rt/L}) \]

Using this expression we can determine the time-dependence of any other circuit parameter.
Since the question asks about the current directly,

\[ i(t) = \frac{V}{R} (1 - e^{-\frac{tR}{L}}) \]

\[ 5 = \frac{100}{15} (1 - e^{-\frac{(0.5)(15)}{L}}) \]

\[ 0.75 = 1 - e^{-\frac{7.5}{L}} \]

\[ e^{-\frac{7.5}{L}} = 0.25 \]

\[ -\frac{7.5}{L} = \ln(0.25) \]

\[ L = 5.4 \text{ H} \]

Therefore, if the electromagnet has an inductance of 5.4 H, it will take 0.5 s for the current to rise to 5.0 A.
Electric Circuits

Activities
The left block below has front face dimensions of 10 cm by 4 cm, with a depth of 3 cm. The right block is made of the same material and is exactly one-half as wide, with front face dimensions of 5 cm by 4 cm, with a depth of 3 cm.

Imagine connecting the two blocks together in either series or parallel using any of the current paths. For example, you could connect the blocks in series such that current flows first in the C direction and then in the E direction.

a. Rank the electrical resistance along each of the hypothetical current paths.

   Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest
   _____ The ranking cannot be determined based on the information provided.

   Explain the reason for your ranking:

b. Consider paths A and D in series. How does this total resistance compare to the resistances along A and D separately? Explain.

c. Consider paths C and F in parallel. How does this total resistance compare to the resistances along C and F separately? Explain.

d. Consider paths B and E in parallel. How does this total resistance compare to the resistances along C and F separately? Explain.
Each of the circuits below consists of identical batteries and resistors. All of the switches are closed at the same time.

a. Rank the circuits on the basis of their total resistance.

Largest 1. 2. 3. 4. 5. 6. Smallest

The ranking cannot be determined based on the information provided.

b. Rank the circuits on the basis of the elapsed time before the battery “dies”.

Largest 1. 2. 3. 4. 5. 6. Smallest

The ranking cannot be determined based on the information provided.

Explain the reasons for your rankings:
Each of the circuits below consists of identical batteries and resistors. All of the switches are closed at the same time.

a. Rank the circuits on the basis of the current through the resistor labeled R.

Largest  1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

b. Rank the circuits on the basis of the magnitude of the potential difference across the resistor labeled R.

Largest  1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reasons for your rankings:
Six air-filled parallel plate capacitors have the different plate areas and capacitances listed below.

<table>
<thead>
<tr>
<th>Area</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4 cm²</td>
</tr>
<tr>
<td>B</td>
<td>4 cm²</td>
</tr>
<tr>
<td>C</td>
<td>2 cm²</td>
</tr>
<tr>
<td>D</td>
<td>8 cm²</td>
</tr>
<tr>
<td>E</td>
<td>1 cm²</td>
</tr>
<tr>
<td>F</td>
<td>2 cm²</td>
</tr>
</tbody>
</table>

a. Rank these capacitors on the basis of the separation between the plates.

Largest  1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest  _____

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

b. All of the capacitors are attached to batteries with the same potential difference. Rank the capacitors on the basis of the charge stored on the positive plate.

Largest  1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest  _____

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
Each of the circuits below consists of identical batteries, resistors, and capacitors. All of the switches are closed at the same time.

a. Rank each circuit on the basis of the time needed for the positive plate of the capacitor to reach 50% of full charge.


The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

In circuits A and E, the capacitor effectively "shorts" the battery and will reach half charge very quickly. In F, the current to the capacitor flows through a parallel circuit, which allows charge to reach the capacitor quickly. In B and C, the current must flow through a single resistor and in D through two resistors in series. This results in a longer time to reach half charge.

b. Rank each circuit on the basis of the final charge on the positive plate of the capacitor.

Largest 1. 2. 3. 4. 5. 6. Smallest

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
Each of the circuits below consists of identical batteries, resistors, and capacitors. All of the switches are closed at the same time.

A

B

C

D

E

F

Rank each circuit on the basis of the final charge on the positive plate of the capacitor.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest _____

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
Each of the circuits below consists of identical batteries, resistors, and capacitors. All of the switches are closed at the same time.

A

B

C

D

E

F

a. Rank each circuit on the basis of the current through the battery just after the switch is closed.

Largest  1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

b. Rank each circuit on the basis of the current through the battery long after the switch is closed.

Largest  1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
Each of the circuits below consists of identical batteries, resistors, and inductors. All of the switches are closed at the same time.

A  B  C  D  E  F

a. Rank each circuit on the basis of the current through the battery just after the switch is closed.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest _____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

b. Rank each circuit on the basis of the current through the battery long after the switch is closed.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest _____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
The cylindrical wire used to form a light bulb filament has radius 3.7 \( \mu \text{m} \) and length 1.7 cm.

Mathematical Analysis

a. If the wire is made of tungsten, what is the resistance of the filament? Tungsten has a resistivity of \( 5.25 \times 10^{-8} \Omega \text{m} \).

b. If the light bulb is connected to a 12 V battery, what is the power converted by the light bulb?

c. If the battery has a total stored charge of 0.5 A hr, and produces a constant potential difference until discharged, how long will the light bulb light?

d. What is the total energy converted by the light bulb?
The rectangular block of iron at right has front face dimensions of 10 cm by 4 cm, with a depth of 3 cm.

Mathematical Analysis

a. Find the resistance of the block along each of the three coordinate directions. Iron has a resistivity of $9.68 \times 10^{-8}$ $\Omega \text{m}$.

b. If the block is connected to a 12 V battery through the direction with the least resistance, what is the current through the block?

c. If the battery has a total stored energy of 2.5 W hr, and produces a constant potential difference until discharged, how long will this current flow?
The circuit at right represents a Halloween decoration with light-up eyes and a spooky sound. The 6 V battery provides current for the $R_1 = 12 \, \Omega$ bulb and the $R_2 = 8 \, \Omega$ speaker.

Mathematical Analysis

a. Determine the magnitude of the potential difference across and the current through each circuit component.

<table>
<thead>
<tr>
<th>Circuit Component</th>
<th>$\Delta V_{\text{across}}$</th>
<th>$I_{\text{through}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>battery</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_2$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. If the battery has a total stored energy of $12.5 \, \text{W hr}$, and produces a constant potential difference until discharged, how long will the decoration function?

c. How much total energy is converted by the bulb?
The circuit at right represents a Halloween decoration with light-up eyes and a spooky sound. The 6 V battery provides current for the $R_1 = 20 \, \Omega$ bulb and the $R_2 = 8 \, \Omega$ speaker.

**Mathematical Analysis**

*a. Determine the magnitude of the potential difference across and the current through each circuit component.*

<table>
<thead>
<tr>
<th>Circuit Component</th>
<th>$\Delta V_{\text{across}}$</th>
<th>$i_{\text{through}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>battery</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_2$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*b. If the battery has a total stored charge of 2.5 A hr, and produces a constant potential difference until discharged, how long will the decoration function?*

*c. What percentage of the total energy is converted by the speaker?*
The circuit at right represents a 12 V car battery and two headlights of 4.1 Ω each.

Mathematical Analysis

a. Determine the magnitude of the potential difference across and the current through each circuit component.

<table>
<thead>
<tr>
<th></th>
<th>$\Delta V_{\text{across}}$</th>
<th>$i_{\text{through}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>battery</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₁</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₂</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. If the battery has a total stored charge of 120 A hr, and produces a constant potential difference until discharged, how long will the bulbs stay lit?

c. How much total energy is converted by bulb #1?
The circuit at right represents a 12 V car battery and two mismatched headlights, \( R_1 = 4.1 \, \Omega \) and \( R_2 = 3.31 \, \Omega \).

Mathematical Analysis

a. Determine the magnitude of the potential difference across and the current through each circuit component.

<table>
<thead>
<tr>
<th></th>
<th>( \Delta V_{\text{across}} )</th>
<th>( i_{\text{through}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>battery</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R_1 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R_2 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. If the battery has a total stored energy of 900 W hr, and produces a constant potential difference until discharged, how long will the bulbs stay lit?

c. What percentage of the total energy is converted by bulb #1?
The circuit at right consists of a 24 V battery and three resistors, $R_1 = 8 \, \Omega$, $R_2 = 4 \, \Omega$, and $R_3 = 6 \, \Omega$.

Mathematical Analysis

a. Determine the magnitude of the potential difference across and the current through each circuit component.

<table>
<thead>
<tr>
<th>Circuit Component</th>
<th>$\Delta V_{\text{across}}$</th>
<th>$i_{\text{through}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>battery</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_3$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. If the battery has a total stored charge of 200 A hr, and produces a constant potential difference until discharged, how long will the circuit function?

c. What percentage of the total energy is converted by resistor #2?
The circuit at right consists of a 24 V battery and three resistors, 
$R_1 = 8 \, \Omega$, $R_2 = 4 \, \Omega$, and $R_3 = 6 \, \Omega$.

**Mathematical Analysis**

*a. Determine the magnitude of the potential difference across and the current through each circuit component.*

<table>
<thead>
<tr>
<th>Component</th>
<th>$\Delta V_{\text{across}}$</th>
<th>$i_{\text{through}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>battery</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_3$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*b. If the battery has a total stored energy of 1200 W hr, and produces a constant potential difference until discharged, how long will the circuit function?*

*c. What percentage of the total energy is converted by resistor #2?*
Imagine a pair of parallel metal plates of surface area $A$ separated by a distance $d$.

**Mathematical Analysis**

$a$. Calculate the capacitance of these parallel plates.

$b$. A medical defibrillator must store (and release) approximately 200 J to shock a fibrillating heart back into its normal rhythm. What size capacitor would be needed if such a device were to be charged using a 12 V car battery?
Imagine a coaxial cable of wire radius $a$ and shell radius $b$.

Mathematical Analysis

a. Calculate the capacitance, per meter, of this coaxial cable.

b. How much electrical energy can be stored, per meter, in a coaxial cable of wire radius 0.8 mm and shell radius 0.7 cm before the air between the wire and shell breaks down? Air electrically breaks down at a potential difference of 3 kV/mm.
Imagine a pair of hollow nested spheres of inner radius $a$ and outer radius $b$.

**Mathematical Analysis**

a. Calculate the capacitance of these nested spheres.

b. How much electrical energy is stored on a single hollow sphere of radius 3.0 cm charged to 20 kV relative to ground? (Hint: The outer sphere can be thought of as very far away.)
Assuming the capacitor is initially uncharged and the switch is closed at time \( t = 0 \) s, the current through the resistor in the simple RC circuit at right can be modeled by:

\[
\begin{align*}
    i_R(t) &= \frac{V}{R} e^{-\frac{t}{RC}}
\end{align*}
\]

Based on this result, determine the following as functions of time.

**Mathematical Analysis**

a. The potential difference across the resistor:

b. The potential difference across the capacitor:

c. The charge on the capacitor:

d. The energy stored in the capacitor:

e. The energy converted by the resistor:
The device at right represents a simplified camera flash circuit. With $V = 3 \, V$ and $C = 5000 \, \mu F$, find $R$ such that the flash has $12 \, mC$ stored after $1.2 \, s$.

Mathematical Analysis
The device at right represents a simplified camera flash circuit. With $V = 3\, V$ and $R = 147\, \Omega$, find $C$ such that the flash reaches 95% of full charge in 1.5 s.

Mathematical Analysis
The device at right represents a simplified camera flash circuit. With $V = 3\, V$ and $R = 147\, \Omega$, find $C$ such that the flash has 75% of its maximum energy stored after $1.9\, s$.

Mathematical Analysis
The device at right represents a simplified camera flash circuit. With $V = 3 \, \text{V}$ and $R = 140 \, \Omega$, find $C$ such that the flash has 15 mJ stored after 1.2 s.

Mathematical Analysis
Consider the circuit at right with an initially uncharged capacitor.

Mathematical Analysis

a. What is the current through $R_1$ and the current through $R_2$ immediately after the switch is first closed?

Immediately after the switch is closed, the capacitor acts as a short across $R_2$. Therefore no current flows through $R_2$ and the effective resistance of the circuit is completely due to $R_1$.

$$i_1 = \frac{V}{R_1}$$

$$i_2 = 0$$

b. What is the current through $R_1$ and the current through $R_2$ long after the switch is closed?

Long after the switch is closed, the capacitor will be fully charged and all current will flow through $R_1$ and $R_2$ in series. Therefore the effective resistance of the circuit is $R_1 + R_2$.

$$i_1 = \frac{V}{R_1 + R_2}$$

$$i_2 = \frac{V}{R_1 + R_2}$$

c. What is the current through $R_1$ and the current through $R_2$ immediately after the switch is opened (after being closed a long time)?

If the switch is now opened, $R_1$ will be removed from the circuit ($i_1 = 0$) and the capacitor will discharge through $R_2$. The voltage across both $C$ and $R_2$ after the switch is opened is equal to the voltage immediately before the switch was opened,

$$V_c = V_2 = i_2 R_2$$

$$V_c = \left( \frac{V}{R_1 + R_2} \right) R_2$$

This voltage is what drives the current through $R_2$, so

$$i_2 = \frac{V_c}{R_2}$$

$$i_2 = \frac{V}{R_1 + R_2} = \frac{V}{R_1 + R_2}$$

Thus the current through $R_2$ doesn’t immediately change when the switch is open.
Consider the circuit at right with an initially uncharged capacitor.

Mathematical Analysis

a. What is the potential difference across $R_1$ and the potential difference across $R_2$ immediately after the switch is first closed?

b. What is the potential difference across $R_1$ and the potential difference across $R_2$ long after the switch is closed?

c. What is the potential difference across $R_1$ and the potential difference across $R_2$ immediately after the switch is opened (after being closed a long time)?
Consider the circuit at right with an initially uncharged capacitor.

Mathematical Analysis

a. What is the potential difference across $R_1$ and the potential difference across $R_2$ immediately after the switch is first closed?

b. What is the potential difference across $R_1$ and the potential difference across $R_2$ long after the switch is closed?

c. What is the potential difference across $R_1$ and the potential difference across $R_2$ immediately after the switch is opened (after being closed a long time)?
Consider the circuit at right with an initially uncharged capacitor.

Mathematical Analysis

a. What is the current through $R_1$ and the current through $R_2$ immediately after the switch is first closed?

b. What is the current through $R_1$ and the current through $R_2$ long after the switch is closed?

c. What is the current through $R_1$ and the current through $R_2$ immediately after the switch is opened (after being closed a long time)?
Imagine a solenoid consisting of $N$ loops of wire, each with radius $R$, wrapped around a hollow core of length $l$.

**Mathematical Analysis**

*a. Calculate the inductance of this solenoid.*

*b. 10 m of wire is wrapped in a single layer around a hollow core of radius 1.0 cm to make a solenoid. The distance between each turn is 0.10 mm. What is the inductance of this solenoid?*
Imagine a coaxial cable of wire radius \(a\) and shell radius \(b\).

**Mathematical Analysis**

\(a\). Calculate the inductance, per meter, of this coaxial cable.

\(b\). How much magnetic energy is stored, per meter, in a coaxial cable of wire radius 0.8 mm and shell radius 0.7 cm if the wire carries 2.0 A?
Imagine an N-turn loop of radius R. When the loop carries current i, calculate the magnetic energy stored in the device.

Mathematical Analysis

a. Assuming the magnetic field is uniform and equal to the value of the field at the center of the loop, calculate the inductance of this N-turn loop.

b. How much current must flow in a 2000-turn loop of radius 10 cm to store 5.0 J of magnetic energy?
Assuming the switch is closed at time $t = 0$ s, the current through the inductor in the simple RL circuit at right can be modeled by:

$$i_L(t) = \frac{V}{R} \left(1 - e^{-\frac{t}{\tau}}\right)$$

Based on this result, determine the following as functions of time.

**Mathematical Analysis**

*a. The potential difference across the resistor:*

*b. The potential difference across the inductor:*

*c. The energy stored in the inductor:*

*d. The energy converted by the resistor:*
The device at right represents a simplified electromagnet. With \( V = 40 \, V \) and \( L = 15 \, H \), find \( R \) such that the current (and magnetic field) reaches 80% of its final value in 2.5 s.

Mathematical Analysis
The device at right represents a simplified electromagnet. With $V = 40 \, V$ and $R = 14.7 \, \Omega$ find $L$ such that the current reaches 2.5 $A$ in 4.0 $s$.

Mathematical Analysis
The device at right represents a simplified electromagnet. With $V = 40\ V$ and $L = 8.3\ H$, find $R$ such that the magnet has 75% of its maximum energy stored after 1.9 s.

Mathematical Analysis
The device at right represents a simplified electromagnet. With $V = 40 \, \text{V}$ and $R = 10 \, \Omega$, find $L$ such that the magnet has 5 J of energy stored after 0.3 s.

**Mathematical Analysis**
Consider the circuit at right.

Mathematical Analysis

a. What is the current through $R_1$ and the current through $R_2$ immediately after the switch is first closed?

b. What is the current through $R_1$ and the current through $R_2$ long after the switch is closed?

c. What is the current through $R_1$ and the current through $R_2$ immediately after the switch is opened (after being closed a long time)?
Mathematical Analysis

a. What is the potential difference across $R_1$ and the potential difference across $R_2$ immediately after the switch is first closed?

Immediately after the switch is closed an emf will be induced in the inductor that prevents any current from flowing through it. Therefore current will flow through $R_1$ and $R_2$ in series,

$$i = \frac{V}{R_1 + R_2}$$

resulting in

$$V_1 = \left(\frac{V}{R_1 + R_2}\right)R_1$$

$$V_2 = \left(\frac{V}{R_1 + R_2}\right)R_2$$

b. What is the potential difference across $R_1$ and the potential difference across $R_2$ long after the switch is closed?

Long after the switch is closed the “back” emf created by the inductor drops to zero and the inductor effectively shorts $R_2$, resulting in the entire potential difference being across $R_1$,

$$V_1 = V$$

$$V_2 = 0$$

c. What is the potential difference across $R_1$ and the potential difference across $R_2$ immediately after the switch is opened (after being closed a long time)?

If the switch is now opened, $R_1$ will be removed from the circuit ($V_1 = 0$). The inductor will respond so that the current through the inductor before the switch was opened

$$i_L = i_1 = \frac{V}{R_1}$$

is held constant. After the switch is opened, this inductor current will be driven through $i_2$. Therefore,

$$V_2 = i_2R_2$$

$$V_2 = \left(\frac{V}{R_1}\right)R_2 = \left(\frac{R_2}{R_1}\right)V$$
Consider the circuit at right.

Mathematical Analysis

a. What is the potential difference across $R_1$ and the potential difference across $R_2$ immediately after the switch is first closed?

b. What is the potential difference across $R_1$ and the potential difference across $R_2$ long after the switch is closed?

c. What is the potential difference across $R_1$ and the potential difference across $R_2$ immediately after the switch is opened (after being closed a long time)?
Consider the circuit at right.

Mathematical Analysis

a. What is the current through $R_1$ and the current through $R_2$ immediately after the switch is first closed?

b. What is the current through $R_1$ and the current through $R_2$ long after the switch is closed?

c. What is the current through $R_1$ and the current through $R_2$ immediately after the switch is opened (after being closed a long time)?