The Magnetic Force

*Concepts and Principles*

**From Magnetic Field to Magnetic Force**

As I mentioned in the previous chapter, only moving charged particles can interact with a magnetic field. Stationary electric charges are completely oblivious to the presence of magnetic fields. The magnitude of the magnetic force on a moving electric charge is given by the relation,

\[ F = qvB \sin \phi \]

where

- \( q \) is the charge on the particle of interest,
- \( v \) the velocity of the particle of interest,
- \( B \) is the net magnetic field at the location of the particle of interest (created by all of the other moving charged particles in the universe),
- \( \phi \) is the angle between the velocity vector and the magnetic field vector,
- and the direction of the force is given by different right-hand callisthenic. To find the direction of the force:
  - point the fingers of your right-hand in the direction of the particle velocity,
  - curl your fingers until they point in the direction of the magnetic field (you may have to rotate your wrist (or break your fingers) to do this),
  - your thumb points in the direction of the force on a positive charge (if the charge is negative, the force is in the opposite direction).
Magnetic Force on a Current-Carrying Wire

In many cases, instead of considering moving electric charges individually we will focus our attention on the collection of moving charges that make up an electric current. Since a current is simply a collection of moving charges, an electric current must feel a magnetic force when it passes through a magnetic field.

The magnitude of the magnetic force on a current-carrying wire is given by the relation,

\[ F = iLB \sin \phi \]

where

- \( i \) is the current in the wire of interest,
- \( L \) is the length of wire in the magnetic field,
- \( B \) is the net magnetic field at the location of the wire (created by all of the other moving charged particles in the universe),
- \( \phi \) is the angle between the direction of the current and the magnetic field vector,
- and the direction of the force is given by the right-hand rule described above. To find the direction of the force:
  - point the fingers of your right-hand in the direction of the electric current,
  - curl your fingers until they point in the direction of the magnetic field (you may have to rotate your wrist (or break your fingers) to do this),
  - your thumb points in the direction of the force on the current-carrying wire.
The Magnetic Force

Analysis Tools

Long, Parallel Wires

Three long, parallel wires are located as shown. Each grid square has width \( a \). Find the net magnetic force per unit length on the top wire.

To solve this problem, first find the net magnetic field at the location of the wire of interest, and then calculate the force that this field creates on the top wire. The magnetic field will be the vector sum of the magnetic field from the left wire and the magnetic field from the right wire.

For the left wire, I’ve indicated the direction of the magnetic field. Remember, with your thumb pointing in the direction of the current (out of the page), the direction in which the fingers of your right hand curl is the direction of the tangent vector (counterclockwise).

The magnitude of the magnetic field from the left wire is:

\[
B_{\text{left}} = \frac{\mu_0 i}{2\pi r}
\]

\[
B_{\text{left}} = \frac{\mu_0 (2i)}{2\pi \sqrt{(2a)^2 + (a)^2}}
\]

\[
B_{\text{left}} = \frac{\mu_0 i}{\pi \sqrt{5a}}
\]

To determine the direction of this field, notice that the magnetic field vector is at the same angle relative to the \( y \)-axis that the line connecting its location to the source current is relative to the \( x \)-axis. This line forms a right triangle with \( \theta \) given by:

\[
\tan \theta = \frac{a}{2a}
\]

\[
\theta = 26.6^\circ
\]
Thus, 

\[ B_{\text{left}} = -\frac{\mu_0 i}{\pi \sqrt{5} a} \sin 26.6 \]

\[ B_{\text{left}} = -\frac{\mu_0 i}{\pi \sqrt{5} a} (0.45) \]

\[ B_{\text{left}} = -0.064 \frac{\mu_0 i}{a} \]

\[ B_{\text{left}} = +\frac{\mu_0 i}{\pi \sqrt{5} a} \cos 26.6 \]

\[ B_{\text{left}} = -\frac{\mu_0 i}{\pi \sqrt{5} a} (0.89) \]

\[ B_{\text{left}} = +0.128 \frac{\mu_0 i}{a} \]

At this point, we could repeat the calculation for the right current, but that would be a waste of time. Based on the symmetry of the situation, the magnetic field from the right current has exactly the same magnitude as that of the left current, but with opposite y-direction and the same x-direction.

Thus, when the two magnetic fields are summed to yield the total magnetic field the y-components will cancel and the x-components will add resulting in:

\[ B_x = -0.128 \frac{\mu_0 i}{a} \]

Now that we know the magnetic field at the location of the top wire, we can calculate the direction of the magnetic force on the top wire. The magnitude of this force (per unit length, L) is:

\[ F = iLB \sin \phi \]

\[ F = (2i)L(0.128 \frac{\mu_0 i}{a}) \sin(90^\circ) \]

\[ F = 0.256 \frac{\mu_0 i^2}{a} \]

Note that the angle between the current (into the page) and the magnetic field (to the left) is 90°.

The direction of this force can be determined by the right-hand rule. If your fingers point into the page (the direction of the current) and you curl them until they point in the direction of the magnetic field (to the right), your thumb has no choice but to point in the +y-direction. Thus, the magnetic force pushes the top wire away from the two bottom wires.

Try to convince yourself that wires carrying currents in opposite directions tend to repel each other, while wires carrying currents in the same direction tend to attract each other.
Protons are injected at \(2.0 \times 10^5 \text{ m/s}\) into a 2000 turn-per-meter solenoid carrying 3.0 A clockwise in the diagram at right. Determine the protons’ orbit radius.

A solenoid is a very useful device for generating a uniform magnetic field. A solenoid consists of a wire carrying current \(i\) wrapped \(N\) times around a hollow core of radius \(R\) and length \(L\). Typically, \(L\) is substantially larger than \(R\) (much larger than illustrated below). Contrast this with a coil of wire, in which \(R\) is typically larger than \(L\).

In a solenoid, the magnetic field inside the core is extremely uniform. In a sense, the small radius and long length “concentrate” the magnetic field within the core leading to an approximately constant value. Again, this contrasts with a coil of wire, in which the field varies at different locations inside and outside the coil.

By using calculus and a few simplifying approximations, it can be shown that the field inside the solenoid is given by:

\[
B = \mu_0 \left(\frac{N}{L}\right)i
\]

\[
B = \mu_0 ni
\]

where \(n\) is the turn density, the number of loops of wire, or turns, per meter.

Therefore, the solenoid described in the problem creates a magnetic field

\[
B = \mu_0 ni
\]

\[
B = (1.26 \times 10^{-6})(2000)(3)
\]

\[
B = 7.56 \times 10^{-3} T
\]

This field is directed into the page since the current flows clockwise.
This magnetic field will create a magnetic force on the proton. When the proton first enters the device the magnetic force will be directed upward, causing the path of the proton to bend upward. As the proton begins to move upward the direction of the magnetic force changes, and when the proton is moving directly upward the force will be to the left. This will cause the proton to bend toward the left. When the proton is moving directly leftward the magnetic force will be directed downward, causing the proton to begin to bend downward. And so on …

Since the magnetic force is always perpendicular to the direction of travel of the proton, the magnetic force causes the proton to make a non-stop left-hand turn! The proton will begin moving in circles due to this force, and since the force has no component along the direction of travel of the proton it does no work on the proton and the proton moves at constant speed. Basically, magnetic fields “steer” charged particles but don’t make them speed up or slow down.

With this in mind, let’s apply Newton’s Second Law to the proton the instant it enters the solenoid:

\[ F = ma \]

\[ qvB \sin \phi = ma \]

Since the proton enters the solenoid traveling in the +x-direction and the magnetic field is directed into the page, the angle between these vectors is 90°. By the right-hand rule, the direction of the magnetic force is toward the center of its circular path, the +y-direction. (Remember that this is a radial acceleration and hence can be expressed as \( v^2/r \).)

\[ qvB \sin 90 = m\left(\frac{v^2}{r}\right) \]

\[ qvB = m\frac{v^2}{r} \]

\[ qB = m\frac{v}{r} \]

\[ r = \frac{mv}{qB} \]

\[ r = \frac{(1.67 \times 10^{-27})(2 \times 10^5)}{(1.6 \times 10^{-19})(7.56 \times 10^{-3})} \]

\[ r = 0.28m \]

The proton will circle at constant speed at this radius.
The Magnetic Force

Activities
For each of the situations below, a charged particle enters a region of uniform magnetic field. Determine the direction of the magnetic force on each particle.

a. 

b. 

c. 

d. 

e. 

f. 

g. 

h. 

i. 

j.
For each of the situations below, a charged particle enters a region of uniform magnetic field and follows the path indicated. Determine the direction of the magnetic field.

a. 

b. 

c. 

d. 

e. 

f. 

g. 

h. 

i. 

j. 

Superimposed on the unit cube below are the velocity vectors of six charged particles.

![Diagram of charged particles](image)

a. If the particles are positively charged and the magnetic field is in the +z-direction, determine the direction of the magnetic force on each particle.
   
   A:  
   B:  
   C:  
   D:  
   E:  
   F:  

b. If the particles are negatively charged and the magnetic field is in the +x-direction, determine the direction of the magnetic force on each particle.
   
   A:  
   B:  
   C:  
   D:  
   E:  
   F:  

c. If the particles are positively charged and the magnetic field is in the -y-direction, determine the direction of the magnetic force on each particle.
   
   A:  
   B:  
   C:  
   D:  
   E:  
   F:
Five equal mass particles enter a region of uniform magnetic field directed into the page. They follow the trajectories illustrated below.

a. Rank these particles on the basis of their initial velocity, assuming they have equal magnitude electric charge.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Smallest _____

The ranking cannot be determined based on the information provided.

b. Rank these particles on the basis of their electric charge, assuming they have equal magnitude initial velocity.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Smallest _____

The ranking cannot be determined based on the information provided.
Six particles, traveling at equal speeds, are in a region of uniform magnetic field. They are moving in the directions indicated.

Rank the particles on the basis of the magnitude of the magnetic force acting on them.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest _____

The ranking can not be determined based on the information provided.
Superimposed on the unit cube below are the six segments of a current-carrying closed circuit.

a. If the magnetic field is in the +z-direction, determine the direction of the magnetic force on each segment.
   - A:
   - B:
   - C:
   - D:
   - E:
   - F:

b. If the magnetic field is in the +x-direction, determine the direction of the magnetic force on each segment.
   - A:
   - B:
   - C:
   - D:
   - E:
   - F:

c. If the current direction is reversed and the magnetic field is in the -y-direction, determine the direction of the magnetic force on each segment.
   - A:
   - B:
   - C:
   - D:
   - E:
   - F:
Determine the direction of the magnetic force on each side of the current-carrying closed circuit and the direction of the net torque on the circuit.

a. 

b. 

c. 

d. 

e. 

f. 
Determine the direction of the net magnetic force on each wire. The wires are long, perpendicular to the page, and carry constant current either out of (+) or into (-) the page.

a.

b.

c.

d.

e.
Determine the direction of the net magnetic force on each wire. The wires are long, perpendicular to the page, and carry constant current either out of (+) or into (-) the page.

a.

b.
Determine the direction of the net magnetic force on each wire. The wires are long, perpendicular to the page, and carry constant current either out of (+) or into (-) the page.

a.

b.
Below are free-body diagrams for three long, parallel wires that lie along a straight line. Determine the relative positions, with correct spacing, of the three wires.

a.

b.
Below are free-body diagrams for three long, parallel wires. Determine the relative positions, with correct spacing, of the three wires.

a. 

b.
For each of the six combinations of electric currents listed below, rank the combinations on the basis of the magnetic force acting on the central wire. Forces pointing to the right are positive. The wires are long and parallel.

\[ \begin{array}{ccc}
   i_1 & i_2 & i_3 \\
   A & 1 A & 1 A & 1 A \\
   B & 1 A & -1 A & 1 A \\
   C & -1 A & 1 A & 1 A \\
   D & 1 A & 2 A & -1 A \\
   E & 2 A & 1 A & 2 A \\
   F & 2 A & 1 A & -2 A \\
\end{array} \]

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest _____

The ranking can not be determined based on the information provided.

Explain the reason for your ranking:
For each of the six combinations of electric currents listed below, rank the combinations on the basis of the magnetic force acting on the central wire. Forces pointing to the right are positive. The wires are long and parallel.

<table>
<thead>
<tr>
<th></th>
<th>$i_1$</th>
<th>$i_2$</th>
<th>$i_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1 A</td>
<td>1 A</td>
<td>1 A</td>
</tr>
<tr>
<td>B</td>
<td>1 A</td>
<td>1 A</td>
<td>2 A</td>
</tr>
<tr>
<td>C</td>
<td>-1 A</td>
<td>1 A</td>
<td>-4 A</td>
</tr>
<tr>
<td>D</td>
<td>2 A</td>
<td>2 A</td>
<td>4 A</td>
</tr>
<tr>
<td>E</td>
<td>2 A</td>
<td>1 A</td>
<td>2 A</td>
</tr>
<tr>
<td>F</td>
<td>2 A</td>
<td>1 A</td>
<td>8 A</td>
</tr>
</tbody>
</table>

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest _____

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
For each of the six combinations of electric currents listed below, the magnetic force acting on the central wire is zero. Rank the combinations on the basis of $i_3$. The wires are long and parallel.

<table>
<thead>
<tr>
<th></th>
<th>$i_1$</th>
<th>$i_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1 A</td>
<td>1 A</td>
</tr>
<tr>
<td>B</td>
<td>1 A</td>
<td>-1 A</td>
</tr>
<tr>
<td>C</td>
<td>-1 A</td>
<td>1 A</td>
</tr>
<tr>
<td>D</td>
<td>2 A</td>
<td>2 A</td>
</tr>
<tr>
<td>E</td>
<td>2 A</td>
<td>1 A</td>
</tr>
<tr>
<td>F</td>
<td>1 A</td>
<td>2 A</td>
</tr>
</tbody>
</table>

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest _____ The ranking can not be determined based on the information provided.

Explain the reason for your ranking:
For each of the six combinations of electric currents listed below, rank the combinations on the basis of the magnetic force acting on the right wire. Forces pointing to the right are positive. The wires are long and parallel.

<table>
<thead>
<tr>
<th></th>
<th>i₁</th>
<th>i₂</th>
<th>i₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1 A</td>
<td>1 A</td>
<td>1 A</td>
</tr>
<tr>
<td>B</td>
<td>1 A</td>
<td>-1 A</td>
<td>1 A</td>
</tr>
<tr>
<td>C</td>
<td>-1 A</td>
<td>1 A</td>
<td>1 A</td>
</tr>
<tr>
<td>D</td>
<td>1 A</td>
<td>2 A</td>
<td>-1 A</td>
</tr>
<tr>
<td>E</td>
<td>2 A</td>
<td>1 A</td>
<td>2 A</td>
</tr>
<tr>
<td>F</td>
<td>2 A</td>
<td>1 A</td>
<td>-2 A</td>
</tr>
</tbody>
</table>

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest _____

The ranking can not be determined based on the information provided.

Explain the reason for your ranking:
For each of the six combinations of electric currents listed below, the magnetic force acting on the left wire is zero. Rank the combinations on the basis of $i_3$. The wires are long and parallel.

<table>
<thead>
<tr>
<th></th>
<th>$i_1$</th>
<th>$i_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1 A</td>
<td>1 A</td>
</tr>
<tr>
<td>B</td>
<td>1 A</td>
<td>-1 A</td>
</tr>
<tr>
<td>C</td>
<td>-1 A</td>
<td>1 A</td>
</tr>
<tr>
<td>D</td>
<td>2 A</td>
<td>2 A</td>
</tr>
<tr>
<td>E</td>
<td>2 A</td>
<td>1 A</td>
</tr>
<tr>
<td>F</td>
<td>1 A</td>
<td>2 A</td>
</tr>
</tbody>
</table>

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest _____

The ranking can not be determined based on the information provided.

Explain the reason for your ranking:
For each of the six combinations of electric currents listed below, rank the combinations on the basis of the magnitude of the magnetic force acting on the stationary proton located at the indicated point.

\[ \begin{array}{c|c|c|c|c|c|c|c|c} & i_1 & i_2 & \text{Currents} & \text{Force} \\
A & 1 \text{ A} & 1 \text{ A} & & \\
B & 1 \text{ A} & -1 \text{ A} & & \\
C & -1 \text{ A} & 1 \text{ A} & & \\
D & 1 \text{ A} & 2 \text{ A} & & \\
E & 2 \text{ A} & 1 \text{ A} & & \\
F & -2 \text{ A} & 1 \text{ A} & & \\
\end{array} \]

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest _____

The ranking can not be determined based on the information provided.

Explain the reason for your ranking:
A pair of long, parallel wires separated by 0.5 cm carry 1.0 A in opposite directions. Find the net magnetic force per unit length on each wire.

Mathematical Analysis

Qualitative Analysis
On the graphic above, sketch the direction of the magnetic force on the requested object(s). Explain why the magnetic force points in this direction.
Four long, parallel wires with spacing 1.5 cm each carry 350 mA. Find the net magnetic force per unit length on each wire.

Qualitative Analysis
On the graphic above, sketch the direction of the magnetic force on the requested object(s). Explain why the magnetic force points in this direction.

Mathematical Analysis
Two long, parallel wires are separated by a distance $2a$ along the horizontal axis. A third long, parallel wire is located at $(x, y) = (0, 2a)$. Find the net magnetic force per unit length on the top wire.

**Qualitative Analysis**

*On the graphic above, sketch the direction of the magnetic force on the requested object. Explain why the magnetic force points in this direction.*

**Mathematical Analysis**
Two long, parallel wires are separated by a distance $2a$ along the horizontal axis. A third long, parallel wire is located at $(x, y) = (0, 2a)$. Find the net magnetic force per unit length on the top wire.

**Qualitative Analysis**

On the graphic above, sketch the direction of the magnetic force on the requested object. Explain why the magnetic force points in this direction.

**Mathematical Analysis**
The square loop of wire at right carries current 300 mA, has edge length 10 cm, and lies in a region of uniform 150 mT magnetic field, in the x-direction. Find the force acting on each side of the loop.

Mathematical Analysis
The square loop of wire at right carries current $i$, has edge length $2a$, and lies in a region of uniform magnetic field, $B$, in the $-z$-direction. Find the force acting on each side of the loop.

Mathematical Analysis
The square loop of wire at right carries current $i$, has edge length $2a$, and lies in a region of uniform magnetic field, $B$, in the $x$-direction. Find the force acting on each side of the loop.

Mathematical Analysis
The long, straight wire at right carries current $i$. The rectangular loop carries current $2i$, has edge lengths $a$ and $2a$, and lies a distance $a$ from the straight wire. Find the net force acting on the loop.

Mathematical Analysis
The long, straight wire at right carries current $i$. The rectangular loop carries current $2i$ and has edge lengths $2a$ and $3a$. The wires are insulated from each other. Find the net force acting on the loop.

Mathematical Analysis
A particle of mass $m$, charge $q$, and velocity $v$ enters the region of uniform magnetic field, $B$, shown at right.

a. Determine the radius of the particle’s path ($R$) as a function of $m$, $q$, $v$, and $B$.

Mathematical Analysis

b. Determine the period of the particle’s orbit ($T$) as a function of $m$, $q$, $v$, and $B$.

Mathematical Analysis
A bubble chamber is a device in which the actual paths of subatomic particles are visible by the trails of ionized particles left in their wake. In such a device, it is noted that an electron traces a 1.3 cm radius circle in a region of 75 mT magnetic field.

a. What is the velocity of the electron?

**Mathematical Analysis**

b. If the same radius path is made by a proton, what is the velocity of the proton?

**Mathematical Analysis**
A bubble chamber is a device in which the actual paths of subatomic particles are visible by the trails of ionized particles left in their wake. In such a device, it is noted that a proton traces a 4.9 cm radius circle when traveling at 0.01c.

a. What is the magnitude of the uniform magnetic field in the bubble chamber?

**Mathematical Analysis**

b. If the magnetic field magnitude was doubled, what would happen to the radius of the proton’s path?

**Mathematical Analysis**
Electrons are injected into a 5,000 turn-per-meter solenoid, shown in cross-section at right. The solenoid current is 1.2 A. The electrons travel along a circular path inside the solenoid.

a. On the diagram above, indicate the direction of the solenoid current.

b. How fast must the electrons be traveling in order to follow a 20 cm radius path?

**Mathematical Analysis**

c. In order to double the radius of the electron’s path, what current is needed?

**Mathematical Analysis**
Protons are injected at 0.30c into a 3,000 turn-per-meter solenoid, shown in cross-section at right. The protons travel along a circular path inside the solenoid.

a. On the diagram above, indicate the direction of the solenoid current.

b. What magnitude current is needed for the protons to follow a 55 cm radius path?

Mathematical Analysis

c. If the current is dropped to one-half of this value, what will be the radius of the protons path?

Mathematical Analysis
The device at right is a velocity selector. By adjusting the uniform electric and magnetic fields in the region between the plates, only particles of specific velocity will exit the device through the hole in the second plate. The electric field, \( E \), is directed downward, and the magnetic field, \( B \), is directed into the page. A beam of particles, each with mass \( m \) and charge \( q \) but with varying velocities, enter the device.

a. Determine the speed of the particles that exit the device (\( v_{\text{selected}} \)) as a function of \( m \), \( q \), \( E \), and \( B \).

Mathematical Analysis

b. Where do particles traveling faster than the speed calculated above strike the second plate?

c. If the magnitude of the magnetic field is increased, where do particles traveling at the speed calculated in (a) strike the second plate?
A beam of protons is injected into the velocity selector at right. The magnetic field is 20 mT directed out of the page.

Mathematical Analysis

a. What magnitude and direction electric field is necessary to select protons at 0.6c?

b. Where do higher velocity protons strike the second plate?

c. If the beam of protons is replaced with a beam of electrons, what happens to electrons traveling at 0.6c?
A beam of singly ionized helium is injected into the velocity selector at right. The electric field is 25 kN/C directed up.

Mathematical Analysis

a. What magnitude and direction magnetic field is necessary to select ionized helium at 0.05c?

b. Would this selector also work with doubly ionized helium at 0.05c?

c. If the beam is replaced with a beam of neutral helium, what happens to neutral helium atoms traveling at 0.05c?
The device at right is a mass spectrometer. A beam of singly ionized atoms is injected into the velocity selector. The velocity selector consists of an electric field, \( E \), directed upward, and a magnetic field, \( B_1 \), directed out of the page. The selected ions then enter a region of uniform magnetic field \( B_2 \), follow a curved path, and strike a collecting plate attached to the interior wall of the device. The ions strike the plate a distance \( D \) from the opening of the velocity selector. Determine the mass of the ion (\( m \)) as a function of its electric charge (\( e \)), \( E \), \( B_1 \), \( B_2 \), and \( D \).

Mathematical Analysis
A beam of singly ionized carbon atoms is injected into the velocity selector. The velocity selector consists of a 22 kN/C electric field directed upward, and a 50 mT magnetic field directed out of the page. The selected ions then enter a region of uniform 500 mT magnetic field.

a. How far from the entry hole do the $^{12}$C ions strike the collecting plate?

Mathematical Analysis

b. How far from the entry hole do the $^{14}$C ions strike the collecting plate?

Mathematical Analysis
A beam of singly ionized uranium atoms is injected into the velocity selector. The velocity selector consists of a 40 kN/C electric field directed upward, and a magnetic field directed out of the page. The selected ions then enter a region of uniform magnetic field equal in magnitude to that within the selector. The distance between the $^{235}\text{U}$ and $^{238}\text{U}$ ions on the collecting plate is 1.5 mm. What is the magnetic field in the selector?

Mathematical Analysis