The Magnetic Force

*Concepts and Principles*

**From Magnetic Field to Magnetic Force**

As I mentioned in the previous chapter, only *moving* charged particles can interact with a magnetic field. Stationary electric charges are completely oblivious to the presence of magnetic fields. The magnetic force on a moving electric charge is given by the relation,

\[ \vec{F} = q \vec{v} \times \vec{B} \]

where

- q is the charge on the particle of interest,
- \( \vec{v} \) the velocity of the particle of interest,
- and \( \vec{B} \) is the net magnetic field at the location of the particle of interest (created by all of the other moving charged particles in the universe).

Thus, the direction of the magnetic force on a moving charge is more complicated than in the analogous case of the electric force. In addition to determining the magnetic field at the location of the particle, you must know its velocity and perform a vector cross-product.
Magnetic Force on a Current-Carrying Wire

In many cases, instead of considering moving electric charges individually we will focus our attention on the collection of moving charges that make up an electric current. In an electric current, consider a small amount of charge, \( dq \), moving with velocity \( v \). This velocity can be represented as:

\[
\vec{v} = \frac{d\vec{s}}{dt}
\]

where \( d\vec{s} \) is the instantaneous displacement of this small collection of charge. Based on this observation, we can calculate the force acting on this small amount of charge by:

\[
d\vec{F} = (dq)\vec{v} \times \vec{B}
\]

\[
d\vec{F} = (dq)(\frac{d\vec{s}}{dt}) \times \vec{B}
\]

\[
d\vec{F} = (\frac{dq}{dt})(d\vec{s}) \times \vec{B}
\]

Since the current in a wire is the amount of charge passing through any cross-section of the wire per second,

\[
d\vec{F} = i(d\vec{s}) \times \vec{B}
\]

Therefore the magnetic force on the entire current-carrying wire is given by the relation,

\[
\vec{F} = \int i d\vec{s} \times \vec{B}
\]

where

- \( i \) is the current in the wire of interest,
- \( d\vec{s} \) is an infinitesimal length of the wire of interest,
- \( \vec{B} \) is the net magnetic field at the location of \( d\vec{s} \) (created by all of the other moving charged particles and currents in the universe),
- and the integral is over the entire length of the wire.
The Magnetic Force

**Analysis Tools**

**Long, Parallel Wires**

Three long, parallel wires are located as shown. Each grid square has width $a$. Find the net magnetic force per unit length on the rightmost wire.

The magnetic force on a current-carrying wire is given by the relation,

$$ \vec{F} = \int id\vec{s} \times \vec{B} $$

However, for this scenario the relationship simplifies. First, the magnetic field from the two source wires is constant along the entire length of the wire of interest. Second, since the wire of interest is straight, $d\vec{s}$ is also constant. Therefore, instead of an integral you have:

$$ \vec{F} = iL \times \vec{B} $$

where $L$ is the total length of the wire of interest.

To solve this problem, first find the net magnetic field at the location of the wire of interest, and then perform the cross-product. The magnetic field will be the vector sum of the magnetic field from the leftmost wire ($\vec{B}_L$) and the magnetic field from the central wire ($\vec{B}_C$).

$$ \vec{B}_L = \frac{\mu_0 i}{2\pi r} \hat{i} $$

$$ \vec{B}_L = \frac{\mu_0 (2i)}{2\pi \sqrt{(4a)^2 + (a)^2}} \left( \frac{-a \hat{i} - 4a\hat{j}}{\sqrt{(4a)^2 + (a)^2}} \right) $$

$$ \vec{B}_L = \frac{2\mu_0 i}{2\pi (17a^2)} (-a \hat{i} - 4a\hat{j}) $$

$$ \vec{B}_L = \frac{\mu_0 i}{17\pi a} (-\hat{i} - 4\hat{j}) $$

$$ \vec{B}_L = \left(-0.0588\hat{i} - 0.235\hat{j}\right) \frac{\mu_0 i}{\pi a} $$

$$ \vec{B}_C = \frac{\mu_0 i}{2\pi r} \hat{i} $$

$$ \vec{B}_C = \frac{\mu_0 (i)}{2\pi \sqrt{(3a)^2 + (a)^2}} \left( \frac{a \hat{i} - 3a\hat{j}}{\sqrt{(3a)^2 + (a)^2}} \right) $$

$$ \vec{B}_C = \frac{\mu_0 i}{2\pi (10a^2)} (a \hat{i} - 3a\hat{j}) $$

$$ \vec{B}_C = \frac{\mu_0 i}{20\pi a} (\hat{i} - 3\hat{j}) $$

$$ \vec{B}_C = (0.05\hat{i} - 0.15\hat{j}) \frac{\mu_0 i}{\pi a} $$
Adding these two contributions together yields

\[ \vec{B} = (-0.0088\hat{i} - 0.385\hat{j}) \frac{\mu_0 i}{\pi a} \]

So the magnetic force on the rightmost wire is:

\[ \vec{F} = i\vec{L} \times \vec{B} \]

\[ \vec{F} = (2i)(L\hat{k}) \times (-0.0088\hat{i} - 0.385\hat{j}) \frac{\mu_0 L}{\pi a} \]

\[ \vec{F} = (0.385\hat{i} - 0.0088\hat{j}) \frac{2\mu_0 i^2 L}{\pi a} \]

\[ \vec{F} = (0.245\hat{i} - 0.0056\hat{j}) \frac{\mu_0 i^2 L}{a} \]

and the force per unit length is

\[ \frac{\vec{F}}{L} = (0.245\hat{i} - 0.0056\hat{j}) \frac{\mu_0 i^2}{a} \]

The force is to the right, and slightly downward. Notice that wires carrying currents in opposite directions tend to repel each other, while wires carrying currents in the same direction tend to attract each other.
Curved Wires

The bent wire at right carries current \( i \), consists of two straight segments of length \( 3L \) and a half-circle of radius \( L \), and lies in a region of uniform magnetic field, \( B \), in the \( +x \)-direction. Find the net force acting on the wire.

To find the total magnetic force on this bent wire, treat the wire as three separate wires. The force on the two straight sections is zero, because the current in these sections is parallel to the field, resulting in no magnetic force. Therefore, the force on the entire wire is equal to the force on the curved section, which will require setting up and evaluating an integral.

The differential element is located at an angle \( \theta \), measured clockwise from the \(-x\)-axis. At this location, \( d\vec{s} \) is directed in the \( +x \)-direction and \( +y \)-direction. This results in:

\[
d\vec{s} = Ld\theta (\sin \hat{\theta} + \cos \hat{\theta})
\]

Using this differential element, the force on the curved section of wire is given by:

\[
\vec{F} = \int id\vec{s} \times \vec{B} = \int_{-\pi/2}^{\pi/2} i (Ld\theta (\sin \hat{\theta} + \cos \hat{\theta})) \times (B\hat{i})
\]

\[
\vec{F} = -iLB\hat{k} \int_{-\pi/2}^{\pi/2} d\theta \cos \theta
\]

\[
\vec{F} = -iLB\hat{k} (\sin(\frac{\pi}{2}) - \sin(-\frac{\pi}{2}))
\]

\[
\vec{F} = -2iLB\hat{k}
\]

The magnetic force will cause the curved end of the wire to sink into the plane of the page.
Protons are injected at $2.0 \times 10^5 \text{ m/s}$ into a 2000 turn-per-meter solenoid carrying 3.0 A clockwise in the diagram at right. Determine the protons’ orbit radius.

A solenoid is a very useful device for generating a uniform magnetic field. A solenoid consists of a wire carrying current $i$ wrapped $N$ times around a hollow core of radius $R$ and length $L$. Typically, $L$ is substantially larger than $R$ (much larger than illustrated below). Contrast this with a coil of wire, in which $R$ is typically larger than $L$.

In a solenoid, the magnetic field inside the core is extremely uniform. In a sense, the small radius and long length “concentrate” the magnetic field within the core leading to an approximately constant value. Again, this contrasts with a coil of wire, in which the field varies at different locations inside and outside the coil.

By using Ampere’s Law, and a few simplifying approximations, it can be shown that the field inside the solenoid is given by:

$$B = \mu_0 \left(\frac{N}{L}\right)i$$

$$B = \mu_0 ni$$

where $n$ is the turn density, the number of loops of wire, or turns, per meter.

Therefore, the solenoid described in the problem creates a magnetic field

$$B = \mu_0 ni$$

$$B = (1.26 \times 10^{-6})(2000)(3)$$

$$B = 7.56 \times 10^{-3} \text{T}$$

This field is directed into the page since the current flows clockwise.
This magnetic field will create a magnetic force on the proton. When the proton first enters the device the magnetic force will be directed upward, causing the path of the proton to bend upward. As the proton begins to move upward the direction of the magnetic force changes, and when the proton is moving directly upward the force will be to the left. This will cause the proton to bend toward the left. When the proton is moving directly leftward the magnetic force will be directed downward, causing the proton to begin to bend downward. And so on...

Since the magnetic force is always perpendicular to the direction of travel of the proton, the magnetic force causes the proton to make a non-stop left-hand turn! The proton will begin moving in circles due to this force, and since the force has no component along the direction of travel of the proton it does no work on the proton and the proton moves at constant speed. Basically, magnetic fields “steer” charged particles but don’t make them speed up or slow down.

With this in mind, let’s apply Newton’s Second Law to the proton the instant it enters the solenoid:

\[ \vec{F} = m\vec{a} \]
\[ q\vec{v} \times \vec{B} = m\vec{a} \]

It enters the solenoid traveling in the +x-direction, and it will accelerate toward the center of its circular path, the +y-direction. (Remember that this radial acceleration can be expressed as \(v^2/r\).)

\[ q(v\hat{i}) \times (-\mu_0 n\hat{k}) = m\left(\frac{v^2}{r}\right)\hat{j} \]
\[ q\nu\mu_0 n\hat{j} = m\frac{v^2}{r}\hat{j} \]
\[ q\nu\mu_0 n = m\frac{v}{r} \]
\[ r = \frac{mv}{q\nu\mu_0 n} \]
\[ r = \frac{(1.67 \times 10^{-27})(2 \times 10^5)}{(1.6 \times 10^{-19})(7.56 \times 10^{-3})} \]
\[ r = 0.28m \]

The proton will circle at constant speed at this radius.
The Magnetic Force

Activities
For each of the situations below, a charged particle enters a region of uniform magnetic field. Determine the direction of the magnetic force on each particle.

a. 

b. 

c. 

d. 

e. 

f. 

g. 

h. 

i. 

j. 

For each of the situations below, a charged particle enters a region of uniform magnetic field and follows the path indicated. Determine the direction of the magnetic field.

a. 

b. 

c. 

d. 

e. 

f. 

g. 

h. 

i. 

j. 

Superimposed on the unit cube below are the velocity vectors of six charged particles.

\[ +\text{y} \]
\[ +\text{z} \]
\[ +\text{x} \]

A
B
C
D
E
F

a. If the particles are positively charged and the magnetic field is in the +z-direction, determine the direction of the magnetic force on each particle.
   A:
   B:
   C:
   D:
   E:
   F:

b. If the particles are negatively charged and the magnetic field is in the +x-direction, determine the direction of the magnetic force on each particle.
   A:
   B:
   C:
   D:
   E:
   F:

c. If the particles are positively charged and the magnetic field is in the -y-direction, determine the direction of the magnetic force on each particle.
   A:
   B:
   C:
   D:
   E:
   F:
Five equal mass particles enter a region of uniform magnetic field directed into the page. They follow the trajectories illustrated below.

a. Rank these particles on the basis of their initial velocity, assuming they have equal magnitude electric charge.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

b. Rank these particles on the basis of their electric charge, assuming they have equal magnitude initial velocity.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
Six particles, traveling at equal speeds, are in a region of uniform magnetic field. They are moving in the directions indicated.

Rank the particles on the basis of the magnitude of the magnetic force acting on them.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest _____

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
Superimposed on the unit cube below are the six segments of a current-carrying closed circuit.

![Diagram of a unit cube with six segments labeled A to F]

a. If the magnetic field is in the +z-direction, determine the direction of the magnetic force on each segment.
   
   A:  
   B:  
   C:  
   D:  
   E:  
   F:  

b. If the magnetic field is in the +x-direction, determine the direction of the magnetic force on each segment.
   
   A:  
   B:  
   C:  
   D:  
   E:  
   F:  

c. If the current direction is reversed and the magnetic field is in the -y-direction, determine the direction of the magnetic force on each segment.
   
   A:  
   B:  
   C:  
   D:  
   E:  
   F:  

Determine the direction of the magnetic force on each side of the current-carrying closed circuit and the direction of the net torque on the circuit.

The forces on the top and bottom segments are zero.

The magnetic forces will make the loop rotate clockwise around the y-axis. Thus the net torque is in the \(-j\) direction.

The forces on the top and bottom segments are zero.
Determine the direction of the net magnetic force on each wire. The wires are long, perpendicular to the page, and carry constant current either out of (+) or into (-) the page.

a. 

b. 

c. 

d. 

e. 

16
Determine the direction of the net magnetic force on each wire. The wires are long, perpendicular to the page, and carry constant current either out of (+) or into (-) the page.

a. 

b. 

c. 

d. 

Below are free-body diagrams for three long, parallel wires that lie along a straight line. Determine the relative positions, with correct spacing, of the three wires.

a.

b.
Below are free-body diagrams for three long, parallel wires. Determine the relative positions, with correct spacing, of the three wires.

a.

b.
For each of the six combinations of electric currents listed below, rank the combinations on the basis of the magnetic force acting on the central wire. Forces pointing to the right are positive. The wires are long and parallel.

```
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>i1</td>
<td>i2</td>
<td>i3</td>
</tr>
<tr>
<td>A</td>
<td>1 A</td>
<td>1 A</td>
</tr>
<tr>
<td>B</td>
<td>1 A</td>
<td>-1 A</td>
</tr>
<tr>
<td>C</td>
<td>-1 A</td>
<td>1 A</td>
</tr>
<tr>
<td>D</td>
<td>1 A</td>
<td>2 A</td>
</tr>
<tr>
<td>E</td>
<td>2 A</td>
<td>1 A</td>
</tr>
<tr>
<td>F</td>
<td>2 A</td>
<td>1 A</td>
</tr>
</tbody>
</table>
```

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative _____

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
For each of the six combinations of electric currents listed below, rank the combinations on the basis of the magnetic force acting on the central wire. Forces pointing to the right are positive. The wires are long and parallel.

<table>
<thead>
<tr>
<th></th>
<th>(i_1)</th>
<th>(i_2)</th>
<th>(i_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1 A</td>
<td>1 A</td>
<td>1 A</td>
</tr>
<tr>
<td>B</td>
<td>1 A</td>
<td>1 A</td>
<td>2 A</td>
</tr>
<tr>
<td>C</td>
<td>-1 A</td>
<td>1 A</td>
<td>-4 A</td>
</tr>
<tr>
<td>D</td>
<td>2 A</td>
<td>2 A</td>
<td>4 A</td>
</tr>
<tr>
<td>E</td>
<td>2 A</td>
<td>1 A</td>
<td>2 A</td>
</tr>
<tr>
<td>F</td>
<td>2 A</td>
<td>1 A</td>
<td>8 A</td>
</tr>
</tbody>
</table>

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative _____

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
For each of the six combinations of electric currents listed below, the magnetic force acting on the central wire is zero. Rank the combinations on the basis of \( i_3 \). The wires are long and parallel.

<table>
<thead>
<tr>
<th></th>
<th>( i_1 )</th>
<th>( i_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1 A</td>
<td>1 A</td>
</tr>
<tr>
<td>B</td>
<td>1 A</td>
<td>-1 A</td>
</tr>
<tr>
<td>C</td>
<td>-1 A</td>
<td>1 A</td>
</tr>
<tr>
<td>D</td>
<td>2 A</td>
<td>2 A</td>
</tr>
<tr>
<td>E</td>
<td>2 A</td>
<td>1 A</td>
</tr>
<tr>
<td>F</td>
<td>1 A</td>
<td>2 A</td>
</tr>
</tbody>
</table>

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative _____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
For each of the six combinations of electric currents listed below, rank the combinations on the basis of the magnetic force acting on the right wire. Forces pointing to the right are positive. The wires are long and parallel.

<table>
<thead>
<tr>
<th></th>
<th>i₁</th>
<th>i₂</th>
<th>i₃</th>
</tr>
</thead>
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<tr>
<td>A</td>
<td>1 A</td>
<td>1 A</td>
<td>1 A</td>
</tr>
<tr>
<td>B</td>
<td>1 A</td>
<td>-1 A</td>
<td>1 A</td>
</tr>
<tr>
<td>C</td>
<td>-1 A</td>
<td>1 A</td>
<td>1 A</td>
</tr>
<tr>
<td>D</td>
<td>1 A</td>
<td>2 A</td>
<td>-1 A</td>
</tr>
<tr>
<td>E</td>
<td>2 A</td>
<td>1 A</td>
<td>2 A</td>
</tr>
<tr>
<td>F</td>
<td>2 A</td>
<td>1 A</td>
<td>-2 A</td>
</tr>
</tbody>
</table>

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative _____

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
For each of the six combinations of electric currents listed below, the magnetic force acting on the left wire is zero. Rank the combinations on the basis of $i_3$. The wires are long and parallel.

<table>
<thead>
<tr>
<th></th>
<th>$i_1$</th>
<th>$i_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1 A</td>
<td>1 A</td>
</tr>
<tr>
<td>B</td>
<td>1 A</td>
<td>-1 A</td>
</tr>
<tr>
<td>C</td>
<td>-1 A</td>
<td>1 A</td>
</tr>
<tr>
<td>D</td>
<td>2 A</td>
<td>2 A</td>
</tr>
<tr>
<td>E</td>
<td>2 A</td>
<td>1 A</td>
</tr>
<tr>
<td>F</td>
<td>1 A</td>
<td>2 A</td>
</tr>
</tbody>
</table>

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative _____

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
For each of the six combinations of electric currents listed below, rank the combinations on the basis of the magnitude of the magnetic force acting on the stationary proton located at the indicated point.

<table>
<thead>
<tr>
<th>i_1</th>
<th>i_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1 A</td>
</tr>
<tr>
<td>B</td>
<td>1 A</td>
</tr>
<tr>
<td>C</td>
<td>-1 A</td>
</tr>
<tr>
<td>D</td>
<td>1 A</td>
</tr>
<tr>
<td>E</td>
<td>2 A</td>
</tr>
<tr>
<td>F</td>
<td>-2 A</td>
</tr>
</tbody>
</table>

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest _____

The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:
Below are six configurations of three long, parallel wires carrying current perpendicular to the page.

A
\[ +i \quad +i \quad +i \]

B
\[ +i \quad -i \quad +i \]

C
\[ +i \quad +i \quad -i \]

D
\[ +i \quad -2i \quad -i \]

E
\[ +2i \quad +i \quad -2i \]

F
\[ +i \quad -i \quad +2i \]

a. Rank these configurations on the magnitude of the magnetic force on the lower-left wire.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest _____

[The ranking cannot be determined based on the information provided.]

b. Rank these configurations on the basis of the angle the magnetic force acting on the lower-left wire makes with the +x-axis. Measure angles counterclockwise from the +x-axis.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest _____

[The ranking cannot be determined based on the information provided.]

Explain the reason for your rankings:
A pair of long, parallel wires separated by 0.5 cm carry 1.0 A in opposite directions. Find the net magnetic force per unit length on each wire.

Qualitative Analysis
On the graphic above, sketch the direction of the magnetic force on each wire.

How should the magnetic force on the left wire compare to the magnetic force on the right wire? Explain.

Mathematical Analysis
Four long, parallel wires with spacing 1.5 cm each carry 350 mA. Find the net magnetic force per unit length on each wire.

Qualitative Analysis
On the graphic above, sketch the direction of the magnetic force on each wire.

How should the magnetic force on the leftmost wire compare to the magnetic force on the rightmost wire? Explain.

Mathematical Analysis
Two long, parallel wires are separated by a distance $2a$ along the horizontal axis. A third long, parallel wire is located at $(x, y) = (0, 2a)$. Find the net magnetic force per unit length on each wire.

**Qualitative Analysis**

On the graphic above, sketch the direction of the magnetic force on each wire.

How should the magnetic force on the left wire compare to the magnetic force on the right wire? Explain.

**Mathematical Analysis**
Four long, parallel wires are arranged on a square of edge length 2a. Find the net magnetic force per unit length on each wire.

Qualitative Analysis
On the graphic above, sketch the direction of the magnetic force on each wire.

How should the magnetic force on the upper-left wire compare to the magnetic force on the lower-right wire? Explain.

Mathematical Analysis
The three long, parallel wires at right are located as shown. Each grid square has width $a$. Find the net magnetic force per unit length on the wire carrying current into the page.

Qualitative Analysis
On the graphic above, sketch the direction of the magnetic force on the wire carrying current into the page.

Mathematical Analysis
The three long, parallel wires at right are located as shown. Each grid square has width \( a \). Find the net magnetic force per unit length on the wire carrying current into the page.

**Qualitative Analysis**

*On the graphic above, sketch the direction of the magnetic force on the wire carrying current into the page.***

**Mathematical Analysis**
The square loop of wire at right carries current 300 mA, has edge length 10 cm, and lies in a region of uniform 150 mT magnetic field in the x-direction. Find the force acting on each side of the loop, the net force on the loop, and the net torque on the loop.

Mathematical Analysis

\( \vec{F}_{\text{top}} \) and \( \vec{F}_{\text{bottom}} \) equal zero since the current is parallel to the field, and thus the cross-product is zero. For the right side,

\[
\vec{F}_{\text{right}} = i \hat{L} \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 10 \times 10^{-2} & 0 \times 10^{-2} & 0 \times 10^{-2} \\ 0 \times 10^{-3} & 150 \times 10^{-3} & 10 \times 10^{-2} \end{vmatrix} = 4.5 \times 10^{-3} \hat{k} \text{N}
\]

and for the left side,

\[
\vec{F}_{\text{left}} = i \hat{L} \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 10 \times 10^{-2} & 0 \times 10^{-2} & 0 \times 10^{-2} \\ 0 \times 10^{-3} & 150 \times 10^{-3} & 10 \times 10^{-2} \end{vmatrix} = -4.5 \times 10^{-3} \hat{k} \text{N}
\]

Thus the net force is zero.

To find the net torque, calculate the torque due to each of the \("\text{side}\) forces using the center of the loop as the origin, and sum.

\[
\vec{\tau} = \vec{r} \times \vec{F}
\]

\[
\vec{\tau}_{\text{right}} = 0.05 \hat{i} \times 4.5 \times 10^{-3} \hat{k} = -2.25 \times 10^{-4} \hat{j} \text{Nm}
\]

\[
\vec{\tau}_{\text{left}} = -0.05 \hat{i} \times -4.5 \times 10^{-3} \hat{k} = 2.25 \times 10^{-4} \hat{j} \text{Nm}
\]

\[
\vec{\tau}_{\text{net}} = -4.5 \times 10^{-4} \hat{j} \text{Nm}
\]
The square loop of wire at right carries current $i$, has edge length $2a$, and lies in a region of uniform magnetic field, $B$, in the $-z$-direction. Find the force acting on each side of the loop, the net force on the loop, and the net torque on the loop.

Mathematical Analysis
The square loop of wire at right carries current $i$, has edge length $2a$, and lies in a region of uniform magnetic field, $B$, in the x-direction. Find the force acting on each side of the loop, the net force on the loop, and the net torque on the loop.

Mathematical Analysis
The wire segment at right carries current \( i \), consists of two straight segments of length \( 2a \) and a quarter-circle of radius \( a \), and lies in a region of uniform magnetic field, \( B \), in the x-direction. Find the net force acting on the wire segment.

Mathematical Analysis
The wire segment at right carries current $i$, consists of two straight segments of length $2a$ and a quarter-circle of radius $a$, and lies in a region of uniform magnetic field, $B$, in the z-direction. Find the net force acting on the wire segment.

Mathematical Analysis
The wire segment at right carries current \( i \), consists of two straight segments of length \( 3a \) and two quarter-circles of radius \( a \), and lies in a region of uniform magnetic field, \( B \), in the \( y \)-direction. Find the net force acting on the wire segment.

Mathematical Analysis
The long, straight wire at right carries current $i$. The rectangular loop carries current $2i$, has edge lengths $a$ and $2a$, and lies a distance $a$ from the straight wire. Find the force acting on each side of the loop and the net force on the loop.

Mathematical Analysis
The long, straight wire at right carries current $i$. The rectangular loop carries current $2i$ and has edge lengths $2a$ and $3a$. The wires are insulated from each other. Find the net force acting on the loop.

Mathematical Analysis
The long, straight wire at right, located at \((x, y) = (-2a, 0)\), carries current \(i\) perpendicular to the page. The square loop of edge length \(2a\), centered at the origin, carries current \(2i\). Find the force acting on each side of the loop and the net force on the loop.

Mathematical Analysis
The long, straight wire at right, located at the origin, carries current $i$ perpendicular to the page. The rectangular loop has edge lengths $2a$ and $3a$ and carries current $2i$. Find the force acting on each side of the loop and the net force on the loop.

Mathematical Analysis
A particle of mass $m$, charge $q$, and velocity $v$ enters the region of uniform magnetic field, $B$, shown at right.

Mathematical Analysis

a. Determine the radius of the particle’s path ($R$) as a function of $m$, $q$, $v$, and $B$.

b. Determine the period of the particle’s orbit ($T$) as a function of $m$, $q$, $v$, and $B$. 


In a bubble chamber, the actual paths of subatomic particles are visible by the trails of ionized particles left in their wake. In such a device, it is noted that an electron traces a 1.3 cm radius circle in a region of 75 mT magnetic field.

Mathematical Analysis

a. What is the velocity of the electron?

b. If the same radius path is made by a proton, what is the velocity of the proton?

c. If both electrons and protons can make the same radii path in the same magnetic field (although traveling at different speeds), how can you (easily) distinguish the path of an electron from the path of a proton?
In a bubble chamber, the actual paths of subatomic particles are visible by the trails of ionized particles left in their wake. In such a device, it is noted that a proton traces a 4.9 cm radius circle in 0.1 μs.

Mathematical Analysis
a. What is the magnitude of the uniform magnetic field in the bubble chamber?

b. If the magnetic field magnitude was doubled, what would happen to the radius of the proton’s path?

c. If the magnetic field magnitude was doubled, what would happen to the period of the proton’s orbit?
Electrons are injected into a 5,000 turn-per-meter solenoid, shown in cross-section at right. The solenoid current is 1.2 A. The electrons travel along a circular path inside the solenoid.

**Qualitative Analysis**
On the diagram above, indicate the direction of the solenoid current.

**Mathematical Analysis**
a. How fast must the electrons be traveling in order to follow a 20 cm radius path?

b. In order to double the radius of the electron’s path, what current is needed?

c. If the current is adjusted to the value in (b), what happens to the period of the electron’s orbit?
Protons are injected at 0.03c into a 30,000 turn-per-meter solenoid, shown in cross-section at right. The protons travel along a circular path inside the solenoid.

**Qualitative Analysis**
*On the diagram above, indicate the direction of the solenoid current.*

**Mathematical Analysis**
*a. What magnitude current is needed for the protons to follow a 25 cm radius path?*

*b. If alpha particles (a bound state of two protons and two neutrons) are injected into the solenoid at the same speed, what is their orbital radius?*

*c. How will the orbital period of the alphas compare to the orbital period of the protons?*
The device at right is a velocity selector. By adjusting the uniform electric and magnetic fields in the region between the plates, only particles of specific velocity will exit the device through the hole in the second plate. The electric field, $E$, is directed downward, and the magnetic field, $B$, is directed into the page. A beam of particles, each with mass $m$ and charge $q$ but with varying velocities, enter the device.

Mathematical Analysis

a. Determine the speed of the particles that exit the device ($v_{selected}$) as a function of $m$, $q$, $E$, and $B$.

b. Where do particles traveling faster than the speed calculated above strike the second plate, above or below the hole?

c. If the magnitude of the magnetic field is increased, where do particles traveling at the speed calculated in (a) strike the second plate, above or below the hole?
A beam of protons is injected into the velocity selector at right. The magnetic field is 20 mT directed out of the page.

Mathematical Analysis

a. What magnitude and direction electric field is necessary to select protons at 0.6c?

b. Where do higher velocity protons strike the second plate, above or below the hole?

c. If the beam is replaced with a beam of electrons traveling at the same speed, do the electrons pass through the hole, strike the plate above the hole, or strike the plate below the hole?
A beam of singly ionized helium is injected into the velocity selector at right. The electric field is 25 kN/C directed up.

Mathematical Analysis

a. What magnitude and direction magnetic field is necessary to select ionized helium at 0.05c?

b. If the beam is replaced with a beam of doubly ionized helium traveling at the same speed, do the doubly ionized helium atoms pass through the hole, strike the plate above the hole, or strike the plate below the hole?

c. If the beam is replaced with a beam of un-ionized helium traveling at the same speed, do the un-ionized helium atoms pass through the hole, strike the plate above the hole, or strike the plate below the hole?
The device at right is a mass spectrometer. A beam of singly ionized atoms is injected into the velocity selector. The velocity selector consists of an electric field, \( E \), directed upward, and a magnetic field, \( B_1 \), directed out of the page. The selected ions then enter a region of uniform magnetic field \( B_2 \), follow a curved path, and strike a collecting plate attached to the interior wall of the device. The ions strike the plate a distance \( D \) from the opening of the velocity selector. Determine the mass of the ion (\( m \)) as a function of its electric charge (\( e \)), \( E \), \( B_1 \), \( B_2 \), and \( D \).

Mathematical Analysis
A beam of singly ionized carbon atoms is injected into the velocity selector. The velocity selector consists of a 22 kN/C electric field directed upward, and a 50 mT magnetic field directed out of the page. The selected ions then enter a region of uniform 500 mT magnetic field.

Mathematical Analysis

a. How far from the entry hole do the $^{12}$C ions strike the collecting plate?

b. How far from the entry hole do the $^{14}$C ions strike the collecting plate?
A beam of singly ionized uranium atoms is injected into the velocity selector. The velocity selector consists of a 40 kN/C electric field directed upward, and a magnetic field directed out of the page. The selected ions then enter a region of uniform magnetic field equal in magnitude to that within the selector. The distance between the $^{235}\text{U}$ and $^{238}\text{U}$ ions on the collecting plate is 1.5 mm. What is the magnetic field in the selector?

Mathematical Analysis