Part I: Limits and Continuity

1. a. The values of \( f(x) \) can be made arbitrarily close to 9 for all \( x \) in some sufficiently small interval containing 4.

b. Answers may vary. Values of \( f(x) \) must approach the same number for \( x \)’s left of 3 as for \( x \)’s right of 3, but that number must not be 7 or else \( f(x) \) would be continuous. One such answer is given below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>2.9</th>
<th>2.99</th>
<th>2.999</th>
<th>3</th>
<th>3.001</th>
<th>3.01</th>
<th>3.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>5.1</td>
<td>5.01</td>
<td>5.001</td>
<td>7</td>
<td>4.999</td>
<td>4.99</td>
<td>4.9</td>
</tr>
</tbody>
</table>

2. \( \lim_{{x \to -3}} f(x) = 1 \neq f(-3) = -7 \) so \( f(x) \) is discontinuous at \( x = -3 \).

\[ \lim_{{x \to 1}} f(x) \] doesn’t exist since \( f(x) \) grows without bound in the positive and negative directions as \( x \) approaches 1 and so \( f(x) \) is discontinuous at \( x = 1 \).

\[ \lim_{{x \to 3}} f(x) = 3 \neq \lim_{{x \to 3^-}} f(x) = -2 \] and so \( \lim_{{x \to 3}} f(x) \) doesn’t exist making \( f(x) \) not continuous at \( x = 3 \). \[ \lim_{{x \to 6}} f(x) = 7 \neq \lim_{{x \to 6}} f(x) = -5 \] and so \( \lim_{{x \to 6}} f(x) \) doesn’t exist making \( f(x) \) not continuous at \( x = 6 \).

3. A rational function is continuous at all \( x \) in its domain, and discontinuous for values of \( x \) for which the denominator is zero. \( x^2 + 3x - 10 = 0 \Rightarrow (x + 5)(x - 2) = 0 \Rightarrow x = -5, 2 \)

The function \( f(x) \) is discontinuous at \( x = -5 \) and \( x = 2 \).

4. a. No, since \( \lim_{{x \to -2}} f(x) = 5 \) but \( \lim_{{x \to 2^-}} f(x) = 7 \) so \( \lim_{{x \to 2}} f(x) \) does not exist, and it must for continuity.

b. No, since \( \lim_{{x \to 1}} g(x) = 2 \) but \( g(1) = 4 \) and they must be equal for continuity.

5. The graph of \( \frac{1}{x+1} \) is given below. It shows a vertical asymptote at \( x = -1 \) where the output grows without bound in the positive and negative directions, and a horizontal asymptote \( y = 0 \). a) does not exist  b) 0
6. a) 0; output approaches 0 from the left of –11  
b) –1; output approaches –1 from the right of –11.  
c) DNE; left and right hand limits are unequal  
d) 0; at x = -11 the output is 0  
e) \(-\infty\); there is a vertical asymptote at x = -3 and the function grows without bound in the negative direction approaching 3 from the left  
f) \(\infty\); there is a vertical asymptote at x = -3 and the function grows without bound in the positive direction approaching –3 from the right  
g) –2; from both sides of x = -1 the output approaches y = -2.  
h) 3; at x = -1, y = 3.

7. (a) 

<table>
<thead>
<tr>
<th>x</th>
<th>3.9</th>
<th>3.99</th>
<th>3.999</th>
<th>4.001</th>
<th>4.01</th>
<th>4.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>.5132</td>
<td>.5013</td>
<td>.5001</td>
<td>.4999</td>
<td>.4988</td>
<td>.4880</td>
</tr>
</tbody>
</table>

Enter f(x) in your calculator and use the table function with the independent variable in ask mode and the dependent variable in automatic mode. Enter the x values and observe the f(x) values. The values for f(x) tend toward the same number from the left and right of x = 4. At the values of x closest to x = 4 on either side f(x) rounds to 0.500

(b) 
\[
\lim_{x \to 4} \frac{\sqrt{x-3} - 1}{x-4} = \lim_{x \to 4} \frac{\sqrt{x-3} - 1}{(\sqrt{x-3}+1)(\sqrt{x-3}+1)} = \lim_{x \to 4} \frac{x-4}{(x-4)(\sqrt{x-3}+1)} = \lim_{x \to 4} \frac{1}{\sqrt{x-3}+1} = \frac{1}{2}
\]

8. Enter f(x) in your calculator and use the table function with the independent variable in ask mode and the dependent variable in automatic mode. Enter the x values and observe the f(x) values. The values for f(x) tend toward the same number from the left and right of x = 1. At the values of x closest to x = 1 on either side f(x) rounds to 0.250

<table>
<thead>
<tr>
<th>x</th>
<th>0.9</th>
<th>0.99</th>
<th>0.999</th>
<th>1</th>
<th>1.001</th>
<th>1.01</th>
<th>1.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>0.2653</td>
<td>0.2515</td>
<td>0.2501</td>
<td>undefined</td>
<td>0.2499</td>
<td>0.2485</td>
<td>0.2360</td>
</tr>
</tbody>
</table>

a. 0.250  
b. 0.250  
c. 0.250

9. Make a table with x values a tenth, a hundredth, and a thousandth greater than 0. Enter f(x) in your calculator and use the table function with the independent variable observe the f(x) values. The values for f(x) differ by less and less as x gets smaller. At the values of x closest to x = 0, f(x) rounds to 0.0. The graph of f(x) confirms this result as the values of y tend to 0 as x tends to 0 from the right.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>0.001</th>
<th>0.01</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>undefined</td>
<td>0.00691</td>
<td>0.04605</td>
<td>0.23103</td>
</tr>
</tbody>
</table>
10. \( \lim_{x \to 2} \frac{2x^3 - 5x^2 + 2x - 3}{x - 2} = 2(2)^3 - 5(2)^2 + 2(2) - 3 = -3 \)

11. \( \lim_{x \to 3} \sqrt{x^2 + 5} = \sqrt{3^2 + 5} = \sqrt{14} \)

12. \( \lim_{x \to \pi} (2 \sin \frac{x}{2} + \cos 2x) = (2 \sin \frac{\pi}{2} + \cos 2\pi) = 2(1) + 1 = 3 \)

13. \( \lim_{x \to -1} \frac{x^2 - 2x - 3}{x + 1} = \lim_{x \to -1} \frac{(x + 1)(x - 3)}{x + 1} = \lim_{x \to -1} (x - 3) = -1 - 3 = -4 \)

14. At \( x = 3 \) the denominator is 0 while the numerator is not and so the function has a vertical asymptote. From the graph we note that \( \lim_{x \to 3^-} \frac{x + 9}{x^2 + 6x - 27} = -\infty \), while \( \lim_{x \to 3^+} \frac{x + 9}{x^2 + 6x - 27} = \infty \). We say the limit does not exist at \( x = 3 \).

15. Since to the right of 1 \( x > 1 \), \( \lim_{x \to 1^+} f(x) = 1^2 = 1 \)

16. \( \lim_{x \to \infty} \frac{4 - 5x - 3x^3}{7x^3 - 9x} = \lim_{x \to \infty} \frac{4 - 5x - 3x^3}{7x^3 - 9x} = \lim_{x \to \infty} \frac{4 - 5x}{7x^3 - 9x} = -\frac{3}{7} \)

17. \( \lim_{x \to \infty} \frac{2x}{\sqrt{3x^2 + 5}} = \lim_{x \to \infty} \frac{2x}{\sqrt{3x^2 + 5}} = \lim_{x \to \infty} \frac{2x}{\sqrt{3 + \frac{5}{x^2}}} = \lim_{x \to \infty} \frac{2}{\sqrt{3 + \frac{5}{x^2}}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \)

18. \( \lim_{x \to \infty} \frac{2x^3 - 4x^2 + 5}{x^3} = \lim_{x \to \infty} \frac{2 - \frac{4}{x} + \frac{5}{x^3}}{1} = \frac{2}{1} = 2 \)

19. \( \lim_{x \to 2^-} \frac{|x - 2|}{2 - x} = \lim_{x \to 2^-} \frac{-(x - 2)}{2 - x} = \lim_{x \to 2^-} \frac{-x + 2}{2 - x} = \lim_{x \to 2^-} \frac{2 - x}{2 - x} = 1 \)
20. \( \lim_{x \to 2^+} \frac{|x - 2|}{2 - x} = \lim_{x \to 2^+} \frac{x - 2}{2 - x} = -1 \)

21. Since the left hand and right hand limits do not agree (see problems 19 and 20), the limit does not exist.

22. At \( x = 1 \) the denominator is zero while the numerator is not and so there is a vertical asymptote at \( x = 1 \). From the graph we see that the output grows without bound in the negative direction from both sides of \( x = 1 \), and so we say the limit is \( -\infty \).

23. At \( x = -2 \) the denominator is zero while the numerator is not and so there is a vertical asymptote at \( x = -2 \). From the graph we see that the output grows without bound in the negative direction as \( x \) approaches \(-2\) from the right, and so we say the right hand limit is \( -\infty \).

24. To the right of zero the graph grows without bound in the negative direction along its vertical asymptote, \( x = 0 \), and so we say the limit is \( -\infty \).

25. (a) \( f(x) = \frac{4 - x^2}{x^2 - x - 2} = \frac{(2-x)(2+x)}{(x-2)(x+1)} = \frac{-1(x-2)(2+x)}{(x-2)(x+1)} = \frac{-2-x}{x+1} \)

\[
\lim_{x \to -1^-} \frac{-2-x}{x+1} = \frac{-2 - (-1)}{-1^+ + 1} = \frac{-1}{0^-} = \infty
\]

\[
\lim_{x \to -1^+} \frac{-2-x}{x+1} = \frac{-2 - (-1)}{-1^+ + 1} = \frac{-1}{0^+} = -\infty
\]

The \( \lim_{x \to -1^+} f(x) = -\infty \) and the \( \lim_{x \to -1^-} f(x) = \infty \).

There is a vertical asymptote at \( x = -1 \).
25 \(b\) \[
\lim_{x \to \infty} \frac{4 - x^2}{x^2 - x - 2} = \lim_{x \to \infty} \frac{4 - x}{x^2} = \lim_{x \to \infty} \frac{4 - 1}{1 - \frac{2}{x}} = \frac{-1}{1} = -1
\]

Since \(f(x)\) is a rational function, both the \(\lim_{x \to \infty} f(x) = -1\) and the \(\lim_{x \to -\infty} f(x) = -1\). There is a horizontal asymptote at \(y = -1\).

**B. Differentiation**

26. \(f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \) Note: other variables such as \(h\) may be substituted for \(\Delta x\). Here

\[
f'(x) = \lim_{\Delta x \to 0} \frac{[2(x + \Delta x)^2 - 3(x + \Delta x) - 1] - [2x^2 - 3x - 1]}{\Delta x}
\]

\[
= \lim_{\Delta x \to 0} \frac{2x^2 + 4x\Delta x + 2\Delta x^2 - 3x - 3\Delta x - 1 - 2x^2 - 3x - 1}{\Delta x}
\]

\[
= \lim_{\Delta x \to 0} \frac{4x\Delta x + 2\Delta x^2 - 3\Delta x}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta x(4x + 2\Delta x - 3)}{\Delta x} = \lim_{\Delta x \to 0} (4x + 2\Delta x - 3) = 4x - 3
\]

27. \(f''(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - 2f(x) + f(x - \Delta x)}{\Delta x} \) \[
= \lim_{\Delta x \to 0} \frac{2(x + \Delta x)^2 - 5(x + \Delta x) + 7 - x^2 + 5x - 7}{\Delta x}
\]

\[
= \lim_{\Delta x \to 0} \frac{2x^2 + 2\Delta x^2 - 5\Delta x + 7 - x^2 + 5x - 7}{\Delta x}
\]

\[
= \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x - 5)}{\Delta x} = \lim_{\Delta x \to 0} (2x + \Delta x - 5) = 2x - 5
\]

28. \(f'(x) = \lim_{h \to 0} \frac{4(x + h)^2 + 3(x + h) - (4x^2 + 3x)}{h} = \lim_{h \to 0} \frac{4x^2 + 8xh + 4h^2 + 3x + 3h - 4x^2 - 3x}{h}
\]

\[
= \lim_{h \to 0} \frac{8xh + 4h^2 + 3h}{h} = \lim_{h \to 0} \frac{h(8x + 4h + 3)}{h} = \lim_{h \to 0} (8x + 4h + 3) = 8x + 3
\]

29. \(y' = 2(3x^2 + e^{-4x})(6x - 4e^{-4x})\)
30. \( y = t^2 + 5t^{-4} \Rightarrow y' = \frac{2}{3}t^{-\frac{1}{3}} - 20t^{-5} = \frac{2}{3\sqrt{t}} - \frac{20}{t^5} \)

31. \( y = 2x^3\left(4x^2 - 3\right)^{\frac{1}{2}} \Rightarrow y' = 2x^3\left(\frac{1}{2}(4x^2 - 3)\right)^{-\frac{1}{2}}(8x) + \left(4x^2 - 3\right)^{\frac{1}{2}}(6x^2) = \frac{8x^4}{\sqrt{4x^2 - 3}} + 6x^2\sqrt{4x^2 - 3} \)

32. \( f'(x) = -3x^2 \sin x + 6x \cos x + 2x \sec^2 x + 2 \tan x \)

33. \( y' = y = \ln \sqrt{x} - \ln(5 - x) = \frac{1}{2} \ln x - \ln(5 - x) \Rightarrow y' = \frac{1}{2x} + \frac{1}{5 - x} \)

34. \( y' = -3x \ln 7 \left(7^{-3x}\right) + 7^{-3x} \)

35. \( g'(x) = 5x^4 \tan x + x^5 \sec^2 x \quad 36. \quad h'(x) = \frac{5 \sin x - 5x \cos x}{\sin^2 x} \)

37. \( f(x) = 2 \sec x (\sec x \tan x) = 2(\sec^2 x) \tan x \)

38. \( f''(x) = \frac{\left(2x^3 + 1\right)(14x - 4) - \left(7x^2 - 4x + 3\right)(6x^2)}{(2x^3 + 1)^2} = \frac{28x^4 - 8x^3 + 14x - 4 - 42x^4 - 24x^3 + 18x^2}{(2x^3 + 1)^2} = -\frac{14x^4 + 16x^3 - 18x^2 + 14x - 4}{(2x^3 + 1)^2} \)

39. \( g'(\theta) = 4 \cos \theta - 40 \sin \theta \)

40. \( \frac{dy}{dx} = \frac{\left(4e^x - 3\right)b^x - 5e^x(4e^x)}{(4e^x - 3)^2} = \frac{20e^{2x} - 15e^x - 20e^{2x}}{(4e^x - 3)^2} = \frac{-15e^x}{(4e^x - 3)^2} \)

41. \( \frac{dy}{dx} = 3x^4 \cdot \frac{1}{x} + 12x^3 \ln x = 3x^4(1 + 4 \ln x) \quad 42. \quad f(x) = -\frac{\sin x}{\cos x} = -\tan x \)

43. \( \frac{dy}{dx} = \frac{15x^2 - 4x}{5x^3 - 2x^2 + 1} \quad 44. \quad \frac{dy}{dx} = \frac{5}{x} (\sin(\ln(x)))^4 \cos(\ln(x)) \)

45. \( g'(x) = g(x) = 4(5x - 12)^2 \Rightarrow g'(x) = -8(5x - 12)^{-3}(5) = -\frac{40}{(5x - 12)^3} \)

46. \( f'(x) = \frac{-8x}{\sqrt{1 - (1 - 4x^2)^2}} = \frac{-8x}{\sqrt{8x^2 - 16x^4}} \)

47. \( \tan y = x \Rightarrow \sec^2 y \frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1 + \tan^2 y} = \frac{1}{1 + \tan^2 (\arctan x)} = \frac{1}{1 + x^2} \)
48. \( \frac{dy}{dx} = \frac{3x^2 - 4xy + 4y^2}{2x^2 - 8xy} \)  

49. \( \frac{dy}{dx} = \frac{3y - 2x}{2y - 3x} \)

C. Graph Analysis

50. Not differentiable at \( x = -7, -2, -1, 2, 5 \). At \( x = -7, -2 \) and \( 2 \) \( f \) is not differentiable because it is not continuous. At \( x = -1 \) there is a sharp turn in the graph, indicating that the left and right hand derivatives are not equal. At \( x = 5 \) there is a vertical tangent line.

51. The table below describes the features of \( f(x) \) and their corresponding implications for the graphs of the first and second derivatives. This information was used to sketch graphs of the first and second derivatives.

<table>
<thead>
<tr>
<th>( f )</th>
<th>Increasing ((-4,-2),(2,9))</th>
<th>Decreasing ((-2,2))</th>
<th>CU ((-0.5,3.5),(6,9))</th>
<th>CD ((-4,-0.5),(3.5,6))</th>
<th>CN (x = -2,2,6)</th>
<th>PI (x = -0.5,3.5,6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f' )</td>
<td>+</td>
<td>-</td>
<td>Increasing</td>
<td>Decreasing</td>
<td>0</td>
<td>Rel. Ext.</td>
</tr>
<tr>
<td>( f'' )</td>
<td>+</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

See graphs next page.

52. a) Decreasing; Negative; Increasing; Positive
   b) Increasing; Positive; Increasing; Positive
   c) Increasing; Positive; Decreasing; Negative
   d) Decreasing; Negative; Decreasing; Negative
53. The graph of \( f' \) is shown below using dashed lines. Each segment is constant and equal to the slope of the corresponding line segment on \( f \). At the points where the slope of \( f \) changes, \( f' \) doesn’t exist since the left and right derivatives are unequal there. Each line segment of \( f' \) is horizontal and so the slope is zero. The graph of \( f'' \) is not included (for sake of clarity), but would be the graph of the line \( y = 0 \) with holes at \((-4,0), (-2,0), \) and \((2,0)\) since \( f' \) doesn’t exist at these values of \( x \).

54. Since the derivative changes from positive to negative at \( x = -4 \), the graph of \( f \) has a relative maximum at \( x = -4 \). Since the derivative changes from negative to positive at \( x = 2 \), the graph of \( f \) has a relative minimum at \( x = 2 \). Since the derivative decreasing until \( x = -1 \) and then increases, the graph of \( f \) is concave down until \( x = -1 \) and is then concave up. The graph of \( f \) will rise from its leftmost point an amount equal to the area under the derivative graph shown up to \( x = -4 \). It will then fall an amount equal to the area under the graph of the derivative from \( x = -4 \) to \( x = 2 \). It then rises an amount equal to the area under the graph of the derivative function shown from \( x = 4 \) to the rightmost point shown. The graph of \( f \) must have the shape shown, but there are infinitely many such graphs. Any vertical shift of the graph shown is a solution.
59. \( f(x) \) has a relative maximum at \( x = a \) since the derivative changes from + to - there; 
\( f(x) \) has a point of inflection at \( x = b \) since the derivative has a relative minimum there; 
\( f(x) \) has a relative minimum at \( x = c \) since the derivative changes from - to + there; 
\( f(x) \) has a vertical tangent or vertical asymptote at \( x = d \) since the derivative has a vertical asymptote there, indicating slopes of \( f \) become infinite approaching \( d \).

60. B 61. C 62. D 63. A

64. **Relative Extrema:** \( f' \) changes from + to – at \( x = 2 \) indicating a relative maximum. 
\( f' \) changes from – to + at \( x = 4 \) indicating a relative minimum.

Since \( f(1) = -1 \), and \( f \) increases by an amount equal to the area under \( f'' \) from \( x = 1 \) to \( x = 2 \), \( f(2) \) is \(-1\) plus the area of a triangle with base and height \( = 1 \), 
\( f(2) = -1 + 0.5 = -0.5 \). **(2, -0.5) is a relative maximum point.**

Since \( f \) decreases by an amount equal to the area under \( f' \) from \( x = 2 \) to \( x = 4 \), 
\( f(4) = -0.5 – \) area of a triangle with base and height of 2, \( f(4) = -0.5 – 2 = -2.5 \). 
**(4, -2.5) is a relative minimum point.**

**Points of Inflection:** \( f' \) increases from \( x = 0 \) to \( x = 1 \) and so is concave up there. 
\( f' \) decreases from \( x = 1 \) to \( x = 3 \) and so is concave down there. From \( x = 3 \) to \( x = 5 \), \( f' \) increases again, and so \( f \) is concave up again. Thus \( f \) has points of inflection at the relative extrema of \( f' \), at \( x = 1 \) and \( x = 3 \).

We know that \( f(1) = -1 \) and so **(1,-1) is a point of inflection.**

We know that \( f(2) = -0.5 \) and \( f \) decreases from \( x = 2 \) to \( x = 3 \) by an amount equal to the area under the graph of \( f'' \) from \( x = 2 \) to \( x = 3 \), a triangle with height 2 and width 1. \( f(3) = -0.5 – 1 = -1.5 \). **(3,-1.5) is the other point of inflection.**

**Endpoints:** 
\( f(0) = f(1) – \) area of triangle from \( x = 0 \) to \( x = 1 \) 
\( f(0) = -1 – 0.5 = -1.5 \) 
\( f(5) = f(4) + \) area of triangle from \( x = 4 \) to \( x = 5 \) 
\( f(5) = -2.5 + 0.5 = -2 \)
Graph for problem 64

D. Applications of Differentiation

65. a. \( \bar{v} = \) slope of secant line drawn = \(-500/5\) 
\( = -100 \text{ km/hr} \)

b. \( m = \) slope of tangent line drawn = \(500/10\)
\( = 50 \text{ km/hr} \)

c. 50 km/hr (since positive, moving away from home.)

66. a. \( \frac{\Delta S}{\Delta t} = \frac{21 - 10.8}{82 - 76} = \frac{10.2}{6} = 1.7 \text{ million subscribers per year} \)

b. \( \frac{dS}{dt} \approx \frac{37.5 - 21}{86 - 82} = \frac{16.5}{4} = 4.125 \text{ million subscribers per year} \)

67. a. On \([2,2.5]\), \( \frac{\Delta s}{\Delta t} = \frac{s(2.5) - s(2)}{.5} = \frac{160(2.5) - 16(2.5)^2 - (160(2) - 16(2)^2)}{.5} = \frac{44}{.5} = 88 \text{ ft/sec} \)

On\([2,2.1]\), \( \frac{\Delta s}{\Delta t} = \frac{s(2.1) - s(2)}{.1} = \frac{160(2.1) - 16(2.1)^2 - (160(2) - 16(2)^2)}{.1} = \frac{9.44}{.1} = 94.4 \text{ ft/sec} \)

On\([2,2.01]\), \( \frac{\Delta s}{\Delta t} = \frac{s(2.01) - s(2)}{.01} = \frac{160(2.01) - 16(2.01)^2 - (160(2) - 16(2)^2)}{.01} = \frac{.9584}{.01} = 95.84 \text{ ft/sec} \)

On\([2,2.001]\), \( \frac{\Delta s}{\Delta t} = \frac{s(2.001) - s(2)}{.001} = \frac{160(2.001) - 16(2.001)^2 - (160(2) - 16(2)^2)}{.001} = \frac{.095984}{.001} = 95.984 \text{ ft/sec} \)

b. 96 ft/sec
68. a) \( v(t) = s'(t) = -32t + 40 \) ft/sec  
   b) \( a(t) = v'(t) = -32 \) ft/sec^2  
   c) \( v_{\text{average}} = \frac{s(3) - s(2)}{3 - 2} = \frac{36 - 76}{1} = -40 \) ft/sec  
   d) \( v(2) = -32(2) + 40 = -24 \) ft/sec  

69. a) \( \bar{v} = \frac{\Delta s}{\Delta t} = \frac{s(3.5) - s(3)}{.5} = \frac{-0.4(3.5)^3 + 9(3.5)^2 - (-0.4(3)^3 + 9(3)^2)}{.5} = \frac{22.9}{.5} = 45.8 \) ft/sec  
   b) \( v(t) = s'(t) = -1.2t^2 + 18t \)  
   c) \( v(2) = -1.2(2)^2 + 18(2) = 31.2 \) ft/sec  
   d) \( v(5) = -1.2(5)^2 + 18(5) = 60 \) ft/sec  

70. \( P'(t) = 0 + \frac{(50 + t^2)2000 - 2000t(2t)}{(50 + t^2)^2} \Rightarrow P'(2) = \frac{(50 + 2^2)2000 - 2000(2)(2)}{(50 + 2^2)^2} \approx 31.55 \) bacteria/hour  

71. a) \( \frac{\Delta y}{\Delta x} = \frac{\frac{1}{3} - \frac{1}{2}}{3 - 1} = \frac{-\frac{1}{6}}{2} = -\frac{1}{3} \)  
   b) \( \frac{dy}{dx} = -x^{-2} \Rightarrow \frac{dy}{dx} \bigg|_{x=2} = -\frac{1}{4} \)  

72. a) \( v(t) = h'(t) = 36 - 12t \) ft/sec  
   b) \( v(t) = 0 \Rightarrow 36 - 12t = 0 \Rightarrow t = 3 \) sec  
   c) \( h(3) = 36(3) - 6(3)^2 = 54 \) ft  
   d) Velocity is \( v(5) = 36 - 12(5) = -24 \) ft/sec; Acceleration is \( a(5) = -12 \) ft/sec^2  

73. a) \( G'(3) \) gives the instantaneous rate at which the amount of water in the tank is changing exactly 3 minutes after it starts to drain.  
   b) \( G'(t) = 24t - 240 \)  
   G'(3) = -168 gallons/min.  
   (Note: \( G'(3) \) is negative since the amount of water in the tank is decreasing)  

74. a) \( V'(t) = 35,000 \ln (.75) (.75^t) \)  
   b) \( V'(5) \approx -2,389.39 \) dollars/year  
   (b) The machine is losing value at the rate of \(-2,389.39\) dollars per year.  

75. \( m = y'(1) \) and \( y' = \frac{2}{x} - 6x \Rightarrow y'(1) = -4 \)  
   \( \frac{y - y_i}{x - x_i} = \frac{y + 3}{4 - 1} \Rightarrow y = -4x + 4 - 3 \Rightarrow y = -4x + 1 \)  

76. \( m = f'(3) \) and \( f' = x^2 - 2 \Rightarrow f'(3) = 7 \)  
   \( \frac{y - y_i}{x - x_i} = \frac{y - 3}{7(3 - x)} \Rightarrow y = 7x - 21 + 3 \Rightarrow \text{Equation of tangent line:} \  
   y = 7x - 18 \)
76. Continued
Enter $f(x)$ and the tangent line $7x-18$ into your calculator and use the table feature in ask mode to fill in the following table:

<table>
<thead>
<tr>
<th>$x$</th>
<th>2.5</th>
<th>2.8</th>
<th>2.9</th>
<th>3.0</th>
<th>3.1</th>
<th>3.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>0.2083</td>
<td>1.7173</td>
<td>2.3297</td>
<td>3.0</td>
<td>3.7303</td>
<td>4.5227</td>
</tr>
<tr>
<td>tangent approx.</td>
<td>-0.5</td>
<td>1.6</td>
<td>2.3</td>
<td>3.0</td>
<td>3.7</td>
<td>4.4</td>
</tr>
</tbody>
</table>

77. $m = g'(2)$ and $g' = \frac{-40}{(5y-12)^3} \Rightarrow g'(2) = \frac{-40}{(10-12)^3} = 5$

$y - y_1 = m(x-x_1) \Rightarrow y - 1 = 5(x-2) \Rightarrow y = 5x - 10 + 1 \Rightarrow y = 5x - 9$

78. Minimize Surface Area, $S = 2\pi r^2 + 2\pi rh$

Volume, $V = \pi r^2 h = 128 \pi \Rightarrow h = \frac{128}{r^2} \Rightarrow S = 2\pi r^2 + 2\pi \left(\frac{128}{r^2}\right) = 2\pi r^2 + \pi \left(\frac{256}{r}\right)$

Domain of $S(r)$ is $(0, \infty)$

$S'(r) = 4\pi r - \frac{256\pi}{r^2} = \frac{4\pi^2 r - 256\pi}{r^2} = 0 \Rightarrow 4\pi^2 r - 256\pi = 0 \Rightarrow r^3 = 64 \Rightarrow r = 4$

The derivative is undefined at $r = 0$, but $0$ is not in the domain of $S(r)$ and so the only critical number is $r = 4$. $S''(r) = 4\pi + \frac{512}{r^3} \Rightarrow S''(4) = 4\pi + \frac{512}{4^3} > 0 \Rightarrow$ a relative minimum at $r = 4$. Since $r = 4$ is the only critical number, the relative minimum is the absolute minimum. $h = \frac{128}{4^2} = 8$ The dimensions that minimize surface area are $r = 4$ ft. & $h = 8$ ft. (Min. Surface Area = $96\pi$ ft$^2$ ≈ 301.59 ft$^2$)

79.

(a) Minimize Surface Area, $S = x^2 + 4xh$

Volume, $V = x^2 h = 864 \Rightarrow h = \frac{864}{x^2} \Rightarrow S = x^2 + 4x \cdot \frac{864}{x^2} = x^2 + \frac{3456}{x}$

Domain of $S(x)$ is $(0, \infty)$

$S'(x) = 2x - \frac{3456}{x^2} = \frac{2x^3 - 3456}{x^2} = 0 \Rightarrow 2x^3 - 3456 = 0 \Rightarrow x^3 = \frac{3456}{2} \Rightarrow x = \sqrt[3]{1728} = 12$

The derivative is undefined at $x = 0$, but 0 is not in the domain of $S(x)$ so $x = 12$ is the only CN in the domain. $S''(x) = 2 + \frac{6912}{x^3} \Rightarrow S''(12) = 2 + \frac{6912}{12^3} > 0 \Rightarrow$ rel. min. at 12.

Since there is only one CN, the relative minimum is also the absolute minimum. At $x = 12$, $h = \frac{864}{12^2} = 6$. The dimensions of the box are 12 in. by 12 in. by 6 in.

(b) $S = 12^2 + 4(12)(6) = 432$ square inches
80. Given \( \frac{dV}{dt} = 2 \text{m}^3 / \text{min} \). Find \( \frac{dh}{dt} \)

\[
V = \frac{1}{3} \pi r^2 h \quad \Rightarrow \quad r = \frac{h}{2} \quad \Rightarrow \quad V = \frac{1}{3} \pi \left( \frac{h}{2} \right)^2 h = \frac{\pi}{12} h^3
\]

\[
\frac{dV}{dt} = \frac{3\pi}{12} h^2 \frac{dh}{dt} \Rightarrow 2m^3 / \text{min.} = \frac{\pi}{4} (3m)^2 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{8m^3}{\pi(9m^2)} = \frac{8}{9\pi} \text{m/ min}
\]

81. Given \( \frac{dr}{dt} = 3 \text{ft} / \text{min} \). Find \( \frac{dA}{dt} \) when \( r = 200 \text{ ft} \).

\[
A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi (200 \text{ ft}) \left( 3 \frac{\text{ft}}{\text{min}} \right) = 1200\pi \frac{\text{ft}^2}{\text{min}}
\]

82. a) \( \frac{dS}{dt} = 8\pi \frac{dr}{dt} \)

b) \( \frac{dS}{dt} = 8\pi (3\text{cm})(5\text{cm/sec}) = 120\pi \text{ cm}^2/\text{sec} \approx 377 \text{ cm}^2/\text{sec} \)

83. \( \frac{dx}{dt} = 2 \text{ ft/sec} \). Find \( \frac{dh}{dt} \). We know \( x^2 + h^2 = 25^2 \Rightarrow 2x \frac{dx}{dt} + 2h \frac{dh}{dt} = 0 \Rightarrow \frac{dh}{dt} = -\frac{x}{h} \frac{dx}{dt} \)

When \( x = 15 \text{ ft} \), \( 15^2 + h^2 = 25^2 \Rightarrow h = \sqrt{625 - 225} = 20 \). \( \frac{dh}{dt} = -\frac{15 \text{ ft}}{20 \text{ ft}} \) \( (2 \text{ ft/sec}) = -1.5 \text{ ft/sec} \)

When \( x = 24 \text{ ft} \), \( 24^2 + h^2 = 25^2 \Rightarrow h = \sqrt{625 - 576} = 7 \). \( \frac{dh}{dt} = -\frac{24 \text{ ft}}{7 \text{ ft}} \) \( (2 \text{ ft/sec}) = -48/7 \text{ ft/sec} \)

\( \approx -6.857 \text{ ft/sec} \)

84. (a) \( f'(x) = 3x^2 - 6x = 3(x - 2) = 0 \Rightarrow x = 0, x = 2 \)

(b)

<table>
<thead>
<tr>
<th>Interval</th>
<th>((-\infty, 0))</th>
<th>((0, 2))</th>
<th>((2, \infty))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>-1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>( \text{Sign } f'(k) )</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>( f )</td>
<td>Inc.</td>
<td>Dec.</td>
<td>Inc.</td>
</tr>
</tbody>
</table>

relative maximum: \((0, 4)\);
relative minimum: \((2, 0)\);
f is increasing on \((\infty, 0)\) and \((2, \infty)\);
f is decreasing on \((0, 2)\)

(c) \( f''(x) = 6x - 6 = 6(x - 1) = 0 \Rightarrow x = 1 \)

<table>
<thead>
<tr>
<th>Interval</th>
<th>((-\infty, 1))</th>
<th>((1, \infty))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>( \text{Sign } f'(k) )</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>( f )</td>
<td>CD</td>
<td>CU</td>
</tr>
</tbody>
</table>

inflection point: \((1, 2)\);
f is concave down on \((-\infty, 1)\)
f is concave up on \((1, \infty)\)
85. 
\[ g'(x) = \frac{(x+5)(2x)-(x^2-9)}{(x+5)^2} = \frac{x^2+10x+9}{(x+5)^2} = 0 \Rightarrow x^2+10x+9 = 0 \Rightarrow (x+1)(x+9) = 0 \Rightarrow x = -1, -9 \]
Note: \( g(x) \) is discontinuous at \( x = -5 \)

<table>
<thead>
<tr>
<th>Interval</th>
<th>(-(\infty), -9)</th>
<th>(-9, -5)</th>
<th>(-5, -1)</th>
<th>(-1, (\infty))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>-10</td>
<td>-6</td>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>Sign ( f'(k) )</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

Relative Max at (-9, -18); Relative Min at (-1, -2); Increasing on (-\(\infty\), -9) and (-1, \(\infty\)); Decreasing on (-9, -5) and (-5, -1)

86. 
\[ f''(x) = 3x^2 - 8x = x(3x-8) = 0 \Rightarrow x = 0, x = 8/3 \]
\[ f'''(0) = -8 < 0 \Rightarrow \text{relative min} \]
\[ f'''(8/3) = 8 > 0 \Rightarrow \text{relative max} \]

<table>
<thead>
<tr>
<th>Interval</th>
<th>(-(\infty), 4/3)</th>
<th>(4/3, (\infty))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Sign ( f'(k) )</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>( f )</td>
<td>CD</td>
<td>CU</td>
</tr>
</tbody>
</table>

relative maximum: (0, 0)
relative minimum: (8/3, -256/27)
inflation point(s): (4/3, -128/27)

E. Antiderivatives and Integration

87. \( \int f(x) \, dx = F(x) + C \) where \( F'(x) = f(x) \); that is, the definite integral of \( f(x) \) is the family of all functions whose derivative is \( f(x) \). These functions will differ from each other by at most a constant.

88. \( \frac{5x^4}{4} - x^3 - 2x^2 + 2x + C \)

89. \( \frac{x^9}{3} + 4e^x - 2x + C \)

90. \( \int (2x^{\frac{-7}{2}} - 10x^{\frac{2}{3}}) \, dx = \frac{2x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{10x^{\frac{5}{3}}}{\frac{5}{3}} = 4\sqrt{x} - 6\sqrt[3]{x^5} + C \)
91. \[ \int \left( 10(x^{\frac{1}{3}} + x^{\frac{1}{5}} - 4x) \right) dx = 10 \ln |x| + \frac{x^\frac{5}{2}}{\frac{5}{2}} - \frac{4x^2}{2} + C = \frac{5x^\frac{5}{2}}{5} - 2x^2 + C = \] 10 \ln |x| + \frac{3\sqrt[3]{x^8}}{8} - 2x^2 + C

92. \[ \int \frac{\cos x}{\cos^2 x} dx = \int \cos x dx = \sin x + C \quad \text{93. } - \cos x - \sin x + C \]

94. \[ u = x^2 + 3 \]
\[ du = 2xdx \]
\[ \int u^3 du = \frac{u^4}{4} + C = \frac{(x^2 + 3)^4}{4} + C \]

95. \[ u = 3x \]
\[ du = 3dx \]
\[ \frac{1}{3} \int e^{3x} 3dx = \frac{1}{3} \int e^u du = \frac{1}{3} e^u + C = \frac{1}{3} e^{3x} + C \]

96. \[ \int_{1}^{4} (3x - 2x^{-2} + 5x^{\frac{1}{2}} + 7) dx = \left( \frac{3x^2}{2} - \frac{2x^{-1}}{-1} + \frac{5x^{\frac{3}{2}}}{\frac{3}{2}} + 7x \right) \] \[ = \left( \frac{3(4)^2}{2} - \frac{2(4)^{-1}}{-1} + \frac{5(4)^{\frac{3}{2}}}{\frac{3}{2}} + 7(1) \right) - \left( \frac{3(1)^2}{2} - \frac{2(1)^{-1}}{-1} + \frac{5(1)^{\frac{3}{2}}}{\frac{3}{2}} + 7(1) \right) \]
\[ = (24 + \frac{4}{3}(5(2^3) + 28) - (\frac{1}{2} + 2 + \frac{2}{3}(5) + 7) = \frac{196}{3} \]

97. \[ \int_{a}^{b} (\frac{x^4}{4} - x^3 - 2x^2 - 5x) dx = \left[ \frac{x^5}{5} - \frac{x^4}{4} - \frac{2x^3}{3} - 5x^2 \right]_{a}^{b} = \left[ \frac{(-2)^5}{5} - \frac{(-2)^4}{4} - \frac{2(-2)^3}{3} - 5(-2)^2 \right] - \left[ \frac{(-a)^5}{5} - \frac{(-a)^4}{4} - \frac{2(-a)^3}{3} - 5(-a)^2 \right] = -66 \]

98. \[ 5 \ln x \int_{1}^{2} = 5 \ln(2) - 5 \ln(1) = 5 \ln(2) \]

99. \[ (\sin \theta + 2 \cos \theta) \int_{a}^{b} = [\sin(\frac{a}{2}) + 2 \cos(\frac{a}{2})] - [\sin(\frac{b}{2}) + 2 \cos(\frac{b}{2})] \]
\[ = \left[ \frac{a}{2} + 2(\frac{a}{2}) \right] - [\frac{b}{2} + 2(\frac{b}{2})] = 1 + \frac{a-b}{2} \]

100. \[ -\cos x \int_{1}^{2} = -\cos(\frac{a}{2}) + \cos(\frac{b}{2}) = -\frac{1}{2} + \frac{2}{2} = -\frac{1+\sqrt{2}}{2} \]
101. \[\int_0^{\ln 4} e^x \, dx - \int_0^{\ln 4} e^{-x} \, dx = e^x \left|_0^{\ln 4} - \int_0^{\ln 4} e^{-x} \, dx = e^{\ln 4} - e^0 - \int_0^{\ln 4} e^{-x} \, dx = 4 - 1 = \int_0^{\ln 4} e^{-x} \, dx = 3 - \int_0^{\ln 4} e^{-x} \, dx \]

\[u = -x, \quad du = -dx \]

\[3 - \int_0^{\ln 4} e^{-x} \, dx = 3 - (-1) \int_0^{\ln 4} e^{-x} \, dx = 3 + \int_0^{\ln 4} e^u \, du = 3 + e^u \left|_0^{\ln 4} \right. = 3 + e^0 - 3 = 3 + e^{\ln 4 - 1} - 1 = 2 + 4^{-1} = 2 + \frac{1}{4} = \frac{9}{4} \]

102. \[u = 2x^3 + 3x - 1 \]
\[du = (6x^2 + 3)dx = 3(2x^2 + 1)dx \]
\[u(3) = 2(3)^3 + 3(3) - 1 = 62 \]
\[u(1) = 2(1)^3 + 3(1) - 1 = 4 \]
\[
\int_{\frac{1}{2}}^{62} \frac{u^2}{6} \, du = \int_{\frac{1}{2}}^{62} \frac{62^2}{6} - \frac{4^2}{6} = 638
\]

103. \[u = 2x \]
\[du = 2dx \]
\[u(0) = 0 \]
\[u(2) = 2(\frac{\pi}{2}) = \pi \]
\[
\frac{1}{2} \int_0^\pi \sin u \, du = \frac{-\cos u}{2} \left|_0^\pi \right. = \frac{-\cos \pi + \cos 0}{2} = \frac{1 + 1}{2} = 1
\]

104. \[u = x^2 + 1 \]
\[du = 2xdx \]
\[u(0) = (0^2) + 1 = 1 \]
\[u(1) = (1^2) + 1 = 2 \]
\[
\frac{1}{2} \int_1^2 u^3 \, du = \frac{1}{2} \cdot \frac{u^4}{4} \left|_1^2 \right. = \frac{2^4 - 1^4}{8} = \frac{15}{8}
\]

105. \[u = \sin x \]
\[du = \cos x \, dx \]
\[u(0) = 0; \quad u(\pi/2) = 1 \]
\[
\int_0^{\pi/2} u^2 \, du = \frac{u^3}{3} \left|_0^{\pi/2} \right. = \frac{1}{3} - \frac{0}{3} = \frac{1}{3}
\]

or \[\int_0^{\pi/2} \sin^2 x \cdot \cos x \, dx = \frac{\sin^3 x}{3} \left|_0^{\pi/2} \right. = \frac{\sin^3 (\pi/2) - \sin^3 (0)}{3} = \frac{1}{3} - \frac{0}{3} = \frac{1}{3} \]
106. \( u = x^2 + 1 \)
\( du = 2x \, dx \)

\[
\begin{align*}
u(0) &= 0^2 + 1 = 1 \\
u(1) &= 1^2 + 1 = 2 \\
\frac{1}{2} \int_{1}^{2} du &= \frac{1}{2} (\ln 2 - \ln 1) = \frac{\ln 2}{2}
\end{align*}
\]

107. Rewrite the integrand without absolute values by using two integrals.
\( x^2 - 4 \geq 0 \Rightarrow x^2 \geq 4 \Rightarrow x \geq 2, x \leq -2 \)

\[
\begin{align*}
\int_{-1}^{2} (x^2 - 4) \, dx + \int_{2}^{4} (x^2 - 4) \, dx = & \left[ \frac{x^3}{3} - 4x \right]_{-1}^{2} + \left[ \frac{x^3}{3} - 4x \right]_{2}^{4} \\
= & \left[ \frac{2^3}{3} - 4(2) \right] + \frac{(-1)^3}{3} - 4(-1) + \frac{4^3}{3} - 4(4) - \frac{(-2)^3}{3} - 4(-2) \\
= & \frac{59}{3}
\end{align*}
\]

108. The limits of integration are the same; the value of the integral is 0.

109. \( \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x \, dx = \frac{4}{\pi} (\cos x) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{4}{\pi} [(-\cos \frac{\pi}{2}) - (-\cos \frac{-\pi}{2})] = \frac{4}{\pi} [0 + \frac{\sqrt{2}}{2}] = \frac{2\sqrt{2}}{\pi} \)

110. \( \frac{1}{3} \int_{-1}^{3} (3x^2 - 4x + 5) \, dx = \frac{1}{4} (x^3 - 2x^2 + 5x) \int_{-1}^{3} = \frac{1}{4} \left[ (3^3 - 2(3)^2 + 5(3)) - ((-1)^3 - 2(-1)^2 + 5(-1)) \right] = 8 \)

111. \( \int_{0}^{\pi/2} (e^x - \sin x) \, dx = (e^x + \cos x) \int_{0}^{\pi/2} = e^{\pi/2} + \cos(\pi/2) - [e^0 + \cos(0)] = e^{\pi/2} + 0 - [1 + 1] = e^{\pi/2} - 2 \)

112. \( \int_{0}^{\frac{\pi}{2}} (x + \cos x) \, dx = \left( \frac{x^2}{2} + \sin x \right) \int_{0}^{\frac{\pi}{2}} = \left[ \frac{\pi^2}{2} + \sin \pi \right] - \left[ 0^2 + \sin 0 \right] = \frac{\pi^2}{2} \)

113. \( \int_{0}^{3} (3-x) \sqrt{x} \, dx = \int_{0}^{3} (3x^{1/2} - x^{3/2}) \, dx = \left( \frac{3x^{3/2}}{3/2} - \frac{x^{5/2}}{5/2} \right) \int_{0}^{3} = \left( 2x^{3/2} - \frac{3}{5} x^{5/2} \right) \int_{0}^{3} = 2(3)^{3/2} - \frac{3}{5}(3)^{5/2} - 0 = 6\sqrt{3} - \frac{18}{5}\sqrt{3} = \frac{12\sqrt{3}}{5} \)

114. \( \Delta x = \frac{b - a}{n} = \frac{2 - \frac{1}{3}}{3} = \frac{1.5}{3} = 0.5 \)

\[
\begin{align*}
\int_{1/2}^{2} x^2 \, dx & = f(1) \cdot (0.5) + f(1.5) \cdot (0.5) + f(2) \cdot (0.5) \\
& = 0.5 [1 + 1.837117 + 4] \\
& = 3.41856
\end{align*}
\]
115.  a) \[ \Delta x = \frac{3 - 0}{3} = 1 \]  
Left Hand Sum = (\( f(0) + f(1) + f(2) \))(1) = (0.5 + 3 + 4.5)(1) = 8

b) Right Hand Sum = (\( f(1) + f(2) + f(3) \))(1) = (3 + 4.5 + 5)(1) = 12.5

c) Since \( f(x) \) is monotone increasing, the left hand sum is an underestimate and the right hand sum an overestimate so the exact area is between these sums. Using the left hand sum approximation the maximum error possible would be the difference between the left and right sums, 12.5 – 8 = 4.5 too small.

116.  \[ \Delta x = \frac{b-a}{n} = \frac{2 - (-1)}{5} = \frac{3}{5} = 0.6 \]

\[
\int_{-1}^{2} \frac{1}{x^2 + 2} \, dx = f(-1) \cdot (0.6) + f(-0.4) \cdot (0.6) + f(0.2) \cdot (0.6) + f(0.8) \cdot (0.6) + f(1.4) \cdot (0.6)
\]

\[
= 0.6 [0.33333 + 0.462963 + 0.490196 + 0.378788 + 0.252525]
\]

\[
= 1.15068
\]

Problems 117 - 121:

a) \[ n > \frac{|f(b) - f(a)|}{0.1} \]

Calculations for \[ \frac{|f(b) - f(a)|}{0.1} \] for each problem are given in the table below:

| Problem | \[ \frac{|f(b) - f(a)|}{0.1} \] |
|---------|--------------------------------|
| 117     | \[ \frac{|(2+1)^{\frac{3}{2}} - (0+1)^{\frac{3}{2}}|}{0.1} \] |
| 118     | \[ \frac{2\sqrt{2^2 + 1} - 0\sqrt{0^2 + 1}}{0.1} \] (2 - 0) |
Answers to parts (a)-(d) problems 117 – 121 given in table below.

<table>
<thead>
<tr>
<th>exercise #</th>
<th>(a)</th>
<th>(b.1)</th>
<th>(b.2)</th>
<th>(c)</th>
<th>(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>LHS</td>
<td>RHS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>117</td>
<td>22</td>
<td>3.09492</td>
<td>3.19310</td>
<td>0.09818 &lt; 0.1</td>
<td>3.14401</td>
</tr>
<tr>
<td>118</td>
<td>90</td>
<td>3.34388</td>
<td>3.44326</td>
<td>0.09938 &lt; 0.1</td>
<td>3.39357</td>
</tr>
<tr>
<td>119</td>
<td>19</td>
<td>1.48487</td>
<td>1.38665</td>
<td>0.09822 &lt; 0.1</td>
<td>1.43576</td>
</tr>
<tr>
<td>120</td>
<td>99</td>
<td>6.88479</td>
<td>6.98448</td>
<td>0.09969 &lt; 0.1</td>
<td>6.93463</td>
</tr>
<tr>
<td>121</td>
<td>20</td>
<td>0.92839</td>
<td>1.02683</td>
<td>0.09844 &lt; 0.1</td>
<td>0.97761</td>
</tr>
</tbody>
</table>

(d) The value of the integral is always between the LHS and RHS because if the function is monotonic increasing, the LHS is an underestimate and the RHS is an overestimate; and if the function is monotonic decreasing, the LHS is an overestimate and the RHS is an underestimate.

122. Average the left and right sums of $\int_0^{30} |v(t)| dt$ with 5 rectangles, $\Delta t = 6$ sec.

Left Sum = $(0 + 31 + 56 + 75 + 88)6 = 1500$ feet
Right Sum = $(31 + 56 + 75 + 88 + 95)6 = 2070$ feet
Average = $(1500 + 2070)/2 = 1785$ ft.

123. $\int_3^9 |v(t)| dt = \int_3^9 65 \left(1 - e^{-0.16t}\right) dt$
124. a) Given \( v(0) = 32 \text{ ft/sec} \), \( s(0) = 48 \text{ ft.} \), and \( a(t) = -32 \text{ ft./sec/sec} \).

\[
v(t) = \int -32 \, dt = -32t + C
\]

\( v(0) = -32(0) + C = 32 \Rightarrow C = 32 \Rightarrow v(t) = -32t + 32 \)

\[
s(t) = \int (-32t + 32) \, dt = -16t^2 + 32t + C
\]

\( s(0) = -16(0)^2 + 32(0) + C = 48 \Rightarrow C = 48 \Rightarrow s(t) = -16t^2 + 32t + 48 \)

b) The ball hits the ground when \( s(t) = 0 \).

\[
s(t) = -16t^2 + 32t + 48 = 0 \Rightarrow t^2 - 2t - 3 = 0 \Rightarrow (t - 3)(t + 1) = 0 \Rightarrow t = 3, -1
\]

Reject the negative time, since our model is for times after \( t = 0 \). The ball hits the ground when \( t = 3 \) seconds.

125.

(a) \( v(t) \)

(b) \( s(t) = \int (3 - t) \, dt = 3t - \frac{t^2}{2} + C \)

\( s(0) = 2 \Rightarrow 3(0) - \frac{0^2}{2} + C = 0 \Rightarrow C = 0 \Rightarrow s(t) = 3t - \frac{t^2}{2} + 2 \)

<table>
<thead>
<tr>
<th>( t )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s(t) = 3t - \frac{t^2}{2} + 2 )</td>
<td>4.5</td>
<td>6</td>
<td>6.5</td>
<td>6</td>
<td>4.5</td>
<td>2</td>
</tr>
</tbody>
</table>

(c) Interval \([0, 2]\)

\[
M(2) = \int_0^2 (3 - t) \, dt = 4 \text{ meters}
\]

Interval \([0, 5]\)

\[
M(5) = \int_0^3 (3 - t) \, dt + \int_3^5 (3 - t) \, dt = \left[ 3t - \frac{t^2}{2} \right]_0^3 + \left[ 3t - \frac{t^2}{2} \right]_3^5
\]

\[
= 6.5 \text{ meters}
\]
Continued

(d) average velocity = \[
\frac{1}{5-0} \int_0^5 (3-t) \, dt = \frac{1}{5} \int_0^5 (3t - \frac{t^2}{2}) \, dt = \frac{1}{5} \left[ \frac{15}{2} \right] = 0.5 \text{ m/sec}.
\]

(e) average speed = \[
\frac{1}{5} \int_0^5 |3-t| \, dt = \frac{1}{5} \left( \int_0^3 (3-t) \, dt + \int_3^5 (3-t) \, dt \right) = \frac{1}{5} \left( \left[ 3t - \frac{t^2}{2} \right]_0^3 + \left[ 3t - \frac{t^2}{2} \right]_3^5 \right) = 1.3 \text{ m/sec}.
\]

126. a) \[u = \frac{1}{2}, \quad du = \frac{1}{2} \, dt\]
\[
\int_0^5 100(2^{\frac{1}{2}})dt = \int_0^5 100(2^{\frac{1}{2}})(\frac{1}{2})dt = 500 \int_0^1 (2^u) \, du = 500 \frac{2^u}{\ln 2} \bigg|_0^1 = 500 \frac{2^1}{\ln 2} - 500 \frac{2^0}{\ln 2} \approx 721.35
\]
To the nearest whole number, 721 people.

b) From 1990 to 1995 the population grew by 721 people.

127. \(\Delta x = 0.5\) in the table with 4 intervals from 1 to 3.

Left Sum = ( \(f(1)+f(1.5)+f(2)+f(2.5)\))0.5 = (14 + 12 + 8 + 3) (0.5) = 18.5

128. a) \(v(t) > 0\) when \(0 < t < 1\)

b) \(v(t) < 0\) when \(1.5 < t < 4\)

c) \(v(t) = 0\) when \(1 < t < 1.5\) and \(4 < t < 5\)

d) Area above the t-axis minus area below it = Area triangle – Area trapezoid
\[
= \frac{1}{2} (1)(4) - \frac{2.5+1}{2} (3) = -3.25 \text{ miles, the difference in her starting and ending positions from time } t = 0 \text{ to time } t = 4.
\]

e) The sum of the areas above and below the t - axis
\[
= \frac{1}{2} (1)(4) + \frac{2.5+1}{2} (3) = 7.25 \text{ miles, the total distance she walked from time } t = 0 \text{ to time } t = 4.
\]
f) Since positive velocities take her away from home, a negative change in position means she is closer to home by 3.25 miles than when she started.
129. a) \[ F(x) = (\frac{e^x}{x} - \ln x) \int = [\frac{e^x}{x} - \ln x] - [\frac{3}{x} - \ln(3)] = \frac{e^x}{x} - \ln x - 9 + \ln(3) \]

b) \[ F(4) = \frac{64}{3} - \ln 4 - 9 + \ln 3 = \ln 3 - \ln 4 + \frac{37}{3}; \]
\[ F(5) = \frac{125}{3} - \ln 5 - 9 + \ln 3 = \ln 3 - \ln 5 + \frac{98}{3}; \]
\[ F(e^2) = \frac{e^6}{3} - \ln e^2 - 9 + \ln 3 = \frac{e^6}{3} - 2 + \ln 3 - 9 = \frac{e^6}{3} + \ln 3 - 11 \]

130. a) \[ \int_{1}^{3} f(x) \ dx = -\int_{1}^{3} f(x) \ dx = -6 \]
\[ \int_{1}^{7} f(x) \ dx = \int_{1}^{3} f(x) \ dx + \int_{3}^{7} f(x) \ dx = 6 + 3 = 9 \]

131. a) By the second part of the Fundamental Theorem of Calculus,
\[ F''(x) = \sin^3 x \]

b) \[ \sin^3 x = 0 \Rightarrow x = \pi \]

c)

<table>
<thead>
<tr>
<th>Interval</th>
<th>((0, \pi))</th>
<th>((\pi, 2\pi))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test k</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Sign (F'(k))</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>(F(x))</td>
<td>Inc.</td>
<td>Dec.</td>
</tr>
</tbody>
</table>

The derivative of \(F\) changes sign at \(x = \pi\) from + to – and so \(F\) has a relative maximum at \(x = \pi\)

d) \[ F''(x) = 3 \sin^2 x \cos x \]

e) \[ F''(x) = 3 \sin^2 x \cos x = 0 \Rightarrow \sin x = 0, \cos x = 0 \Rightarrow x = \frac{\pi}{2}, \pi, \frac{3\pi}{2} \]

<table>
<thead>
<tr>
<th>Interval</th>
<th>((0, \pi/2))</th>
<th>((\pi/2, \pi))</th>
<th>((\pi, 3\pi/2))</th>
<th>((3\pi/2, 2\pi))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test k</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Sign (F(k))</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>(F(x))</td>
<td>CU</td>
<td>CD</td>
<td>CU</td>
<td>CD</td>
</tr>
</tbody>
</table>

\(F\) has inflection points at \(x = \pi/2\) and \(x = 3\pi/2\)
131. f) \( F(\pi) = \int_{0}^{\pi} \sin^3 t \, dt \approx \text{MATH 9 fnInt((sin(x))^3,x,0, \pi}) = 1.33 \)

Relative Maximum \( (\pi, 1.33) \)

\( F(\pi/2) = \int_{0}^{\pi/2} \sin^3 t \, dt \approx \text{MATH 9 fnInt((sin(x))^3,x,0, \pi/2}) = .67 \)

\( F(3\pi/2) = \int_{0}^{3\pi/2} \sin^3 t \, dt \approx \text{MATH 9 fnInt((sin(x))^3,x,0, 3\pi/2}) = .67 \)

Points of Inflection: \((\pi/2, 0.67)\) and \((3\pi/2, 0.67)\)

g) \( F(0) = \int_{0}^{0} \sin^3 t \, dt = 0 \)

\[ \begin{align*}
\text{relative maximum} &\quad (\pi, 1.33) \\
\text{Pl's} &\quad \left(\frac{\pi}{2}, .67\right), \left(\frac{3\pi}{2}, .67\right)
\end{align*} \]

h) 

\section*{F. Miscellaneous}

132. \( \int_{a}^{b} f(x) \, dx = \lim_{\|\Delta x\| \to 0} \sum_{i=1}^{n} f(c_i) \Delta x_i \)

133. I. If \( f(x) \) is continuous on \([a,b]\), then \( \int_{a}^{b} f(x) \, dx = F(b) - F(a) \) where \( F'(x) = f(x) \).

II. If \( f(x) \) is continuous on an open interval \( I \) containing \( a \), then, for every \( x \) in \( I \),

\[ \frac{d}{dx} \left[ \int_{a}^{x} f(t) \, dt \right] = f(x) \]

134. If \( f(x) \) is continuous on the closed interval \([a,b]\), then there exists a number \( c \) in the closed interval \([a,b]\) such that \( \int_{a}^{b} f(x) \, dx = f(c)(b - a) \).
135. a) \[ dy = y' \, dx = (3x^2 + 1) \, dx = (3(1)^2 + 1)(.02) = 0.08; \]
\[ \Delta y = y(1.02) - y(1) = (1.02)^3 + 1.02 - (1^3 + 1) = 0.081208 \]

b) \[ dy = y' \, dx = \frac{1}{2\sqrt{x}} \, dx = \frac{1}{2\sqrt{25}} (-0.1) = -0.01; \]
\[ \Delta y = y(24.9) - y(25) = \sqrt{24.9} - \sqrt{25} \approx -0.01001 \]

136. \( \ln(1) = 0. \) \( dy = \frac{1}{x} \, dx. \) \( \ln(1.05) \approx \ln 1 + dy = 0 + \frac{1}{1}(.05) = 0.05 \)