Example 1: A population is normally distributed with mean $\mu = 80$ and standard deviation $\sigma = 10$. If a random sample of size 16 is selected from this population, what is the probability the sample mean $\bar{x}$ will lie between 76 and 84?

Example 2: The heights of female students at a certain university are normally distributed and have a mean $\mu = 64$ inches and a standard deviation $\sigma = 2.5$ inches. If a random sample of size 25 is selected to make an intramural basketball squad, what is the probability that the mean height of the squad will be between 63 and 65 inches?
Answers

1. In this problem $\mu_x = \mu = 80$ and $\sigma_x = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{16}} = \frac{10}{4} = 2.5$

For the boundary on the left $\bar{x} = 76$, and $z = \frac{\bar{x} - \mu_x}{\sigma_x} = \frac{76 - 80}{2.5} = \frac{-4}{2.5} = -1.6$

For the boundary on the right $\bar{x} = 84$, and $z = \frac{\bar{x} - \mu_x}{\sigma_x} = \frac{84 - 80}{2.5} = \frac{4}{2.5} = 1.6$

The area from $z = -1.6$ to $z = 0$ is $.4452$ and the area from $z = 0$ to $z = 1.6$ is also $.4552$ and the SDSM looks like:

So now we know that $P(76 < \bar{x} < 84) = P(-1.6 < z < 1.6) = .4452 + .4452 = .8904$

**Single sample interpretation:** The probability that the $\bar{x}$ from this one random sample will lie between 76 and 84 is .8904.

**SDSM interpretation:** If many random samples of size 16 are selected from this population, we would expect 89.04% of them to have a mean, $\bar{x}$, that lies between 76 and 84.
2. In this problem $\mu_x = \mu = 64$ and $\sigma_x = \frac{\sigma}{\sqrt{n}} = \frac{2.5}{\sqrt{25}} = \frac{2.5}{5} = 0.5$

For the boundary on the left $\bar{x} = 63$, and
$$z = \frac{\bar{x} - \mu_x}{\sigma_x} = \frac{63 - 64}{0.5} = \frac{-1}{0.5} = -2$$

For the boundary on the right $\bar{x} = 65$, and
$$z = \frac{\bar{x} - \mu_x}{\sigma_x} = \frac{65 - 64}{0.5} = \frac{1}{0.5} = 2$$

The area from $z = -2$ to $z = 0$ is .4772 and the area from $z = 0$ to $z = 2$ is also .4772 and the SDSM looks like:

So now we know that $P(63 < \bar{x} < 65) = P(-2 < z < 2) = .4772 + .4772 = .9544$

**Single sample interpretation:** The probability that the $\bar{x}$ from this one random sample will lie between 63 and 65 is .9544.

**SDSM interpretation:** If many random samples of size 25 are selected from this population, we would expect 95.44% of them to have a mean, $\bar{x}$, that lies between 63 and 65.