The Pythagorean Theorem

The **Pythagorean Theorem** expresses the relationship between the two legs (labeled $a$ and $b$) of a right triangle and the hypotenuse (labeled $c$).

The side $c$ is the longest side and is always opposite the right angle.

![Diagram of a right triangle with labels a, b, and c]

The Pythagorean Theorem states that the square of the hypotenuse is equal to the sum of the squares of the legs. In symbols, we write:

$$c^2 = a^2 + b^2$$

**Example 1:** In the triangle below, $a = 3$ and $b = 4$. Determine the side $c$.

![Diagram of a right triangle with labels a=3, b=4, and c]

\[ c^2 = a^2 + b^2 \]
\[ c^2 = 3^2 + 4^2 \]
\[ c^2 = 9 + 16 \]
\[ c^2 = 25 \]
\[ c = \sqrt{25} = 5 \]
Example 2: In the triangle below, \(a = 4\) and \(b = 7\). Determine the side \(c\).

\[
c^2 = a^2 + b^2 \\
c^2 = 4^2 + 7^2 \\
c^2 = 16 + 49 \\
c^2 = 65 \\
c = \sqrt{65} \approx 8.062
\]

Example 3: In the triangle below, \(a = 3\) and \(c = 10\). Determine the side \(b\).

\[
c^2 = a^2 + b^2 \\
10^2 = 3^2 + b^2 \\
100 = 9 + b^2 \\
91 = b^2 \quad \text{or} \\
b^2 = 91 \\
b = \sqrt{91} \approx 9.539
\]

Exercises

1. In a right triangle \(a = 5, b = 8\), use the Pythagorean Theorem to find \(c\).

2. In a right triangle \(c = 12, a = 6\), use the Pythagorean Theorem to find \(b\).

3. In a right triangle, \(b = 10, c = 24\), use the Pythagorean Theorem to find \(a\).

Answers  
1. \(c = 9.43\)  
2. \(b = 10.39\)  
3. \(a = 21.81\)