Consider the probability distribution formula \( P(x) = \frac{x^2 - 2x + 3}{20} \), where the random variable \( x = 2, 3, \) or \( 4. \)

1. Using this formula, calculate the three probabilities associated with the given values of the random variable \( x. \)

2. Using your answers to part 1, create the probability distribution table.

3. Using your answer to part 2, create a probability histogram for the random variable \( x. \)

4. Using the table from part 2, determine the population mean \( \mu \) and the population standard deviation \( \sigma \) of this probability distribution.
Answers

1. Using this formula, calculate the three probabilities associated with the given values of the random variable x.

   If \( x = 2 \), \( P(x) = \frac{x^2 - 2x + 3}{20} = \frac{4 - 4 + 3}{20} = \frac{3}{20} = 0.15 \)

   If \( x = 3 \), \( P(x) = \frac{x^2 - 2x + 3}{20} = \frac{9 - 6 + 3}{20} = \frac{6}{20} = 0.30 \)

   If \( x = 4 \), \( P(x) = \frac{x^2 - 2x + 3}{20} = \frac{16 - 8 + 3}{20} = \frac{11}{20} = 0.55 \)

2. Using your answers to part 1, create the probability distribution table.

   \[
   \begin{array}{c|c}
   x & P(x) \\
   \hline
   2 & 0.15 \\
   3 & 0.30 \\
   4 & 0.55 \\
   \end{array}
   \]

3. Using your answer to part 2, create a probability histogram for the random variable x.
4. Using the table from part 2, determine the population mean $\mu$ and the population standard deviation $\sigma$ of this probability distribution.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$P(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>0.30</td>
</tr>
<tr>
<td>4</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Using this table we can write:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$P(x)$</th>
<th>$xP(x)$</th>
<th>$x^2P(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.15</td>
<td>0.30</td>
<td>0.60</td>
</tr>
<tr>
<td>3</td>
<td>0.30</td>
<td>0.90</td>
<td>2.7</td>
</tr>
<tr>
<td>4</td>
<td>0.55</td>
<td>2.2</td>
<td>8.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.4</td>
<td>12.1</td>
</tr>
</tbody>
</table>

So, $\mu = \sum x \cdot p(x) = 3.4$ (this is the mean of the probability distribution).

And, $\sigma^2 = \sum x^2 \cdot P(x) - \mu^2 = 12.1 - (3.4)^2 = 12.1 - 11.56 = 0.54$

So, $\sigma = \sqrt{\sigma^2} = \sqrt{0.54} = 0.7348469 \approx 0.73$ (this is the standard deviation of the probability distribution).