A social worker wants to know whether the mean annual salary of her clients matches the mean annual salary for all city residents, which is $28,000, or whether it is less. She obtains a random sample of her clients salaries of size 36. The mean of the sample is $27,500 and the standard deviation is $1,200. Perform a hypothesis test at the 5% level of significance.
Solution

\( H_0: \mu = \$28000 \)

\( H_a: \mu < \$28000 \)

The critical value of \( t \) for a one-tailed test on the left is \( t_{(df, \alpha)} = t(35, .05) = -1.69 \)

\[
t^* = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{27,500 - 28,000}{1200/\sqrt{36}} = -500/200 = -2.5
\]

This value of \( t^* \) lies in the rejection region for \( H_0 \), so we must reject \( H_0 \). Based on this sample using the t-distribution with a level of significance equal to .05 and a one-tailed test on the left it seems as though the mean salary of the social workers’ clients is less than the city average. The sample mean of $27,500 is not due to random chance.