Example 2: For years the average tire lifetimes of tires on city police cars was 25,000 miles. Over the last few years city mechanics have begun to suspect that the mean lifetime is no longer as big as it was. They will perform a hypothesis test by selecting a random sample of 40 new tires, which will be put into service and observed over time. The mayor wants the test to be performed at the 2% level of significance (that is, $\alpha = .02$). Suppose that after all the data is collected from this sample of 40 tires that $\bar{x} = 24,200$ miles. What decision should be made? (Assume that $\sigma = 2000$ miles.)
Solution

(1) \( H_0: \ \mu = 25,000 \)

\( H_a: \ \mu < 25,000 \)

(2) & (3) The SDSM is normally distributed with mean \( \mu_x = \mu = 25,000 \) and

\[
\sigma_x = \frac{\sigma}{\sqrt{n}} = \frac{2000}{\sqrt{40}} \approx 316.22777 \approx 316
\]

(4) The test statistic for the sample is

\[
z^* = \frac{\bar{x} - \mu_x}{\sigma_x} = \frac{24,200 - 25,000}{316.22777} \approx -2.53
\]

(5) By comparing the value of the test statistic \( z^* \) with the value of \( z_{\text{critical}} \) we see that \( z^* \) lies in the Rejection Region for \( H_0 \). So, we will reject \( H_0 \).

(6) Based on this one sample, with a one-tailed test on the left, at the 2% level of significance (that is, \( \alpha = .02 \)), we will reject \( H_0 \). It seems that the real mean lifetime of tires is less than 25,000 miles.