When making a $1 - \alpha$ confidence interval estimate for the population mean $\mu$ when $\sigma$ is unknown, the lower confidence limit is:

$$\bar{x} - \frac{t(df, \alpha/2)s}{\sqrt{n}}$$

and the upper confidence limit is $\bar{x} + \frac{t(df, \alpha/2)s}{\sqrt{n}}$, where $n = $ size of sample used to estimate $\mu$

$df = n - 1$

$\bar{x} = $ mean of sample and is the point estimate of $\mu$

$s = $ standard deviation of the sample is the point estimate of $\sigma$

$-t(df', \alpha/2) = $ lower confidence limit for the t-distribution

$+t(df', \alpha/2) = $ upper confidence limit for the t-distribution

**Example:** We want to determine a 98% confidence interval estimate for the number of miles driven in a week by MCC students who are car owners.

We select a random sample of size 51. Since we have no information about the shape of the distribution of miles and since $n \geq 30$, we can use the t-distribution.

Suppose that the sample mean $\bar{x} = 140$ miles and the sample standard deviation $s = 11.5$ miles.
Solution

Since we want the confidence to be 98%, that means \( \alpha = 2\% = 0.02 \). So, \( \alpha/2 = 0.01 \) is in each tail of the t-distribution.

The error of the estimate \( E = \frac{t(50, .01)s}{\sqrt{n}} = \frac{(2.40)(11.5)}{\sqrt{51}} = \frac{27.6}{7.141428429} = 3.86477 \approx 4 \)

So our lower confidence limit is \( LCL = 140 - 4 = 136 \) miles,

and our upper confidence limit is \( UCL = 140 + 4 = 144 \) miles

So, based on this one sample we can say that we are 98% confident that the real population mean \( \mu \) (miles driven per week by MCC students who are car owners) lies between 136 miles and 144 miles.

In symbols, \( P(136 < \mu < 144) = 0.98 \). The picture below summarizes what we have done.

Note: The probability of \( \mu \) NOT lying between 136 and 144 is 2% = 0.02, which means that we can think of \( \alpha \) as being the probability that we have made an error in our estimate of \( \mu \).