Exposing Students to Statistical Methods Based on Counts and Ranks

Beyond The Formula VIII
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Getting the ball rolling
Why Study Alternative Methods for Inference?

- These procedures are applicable to data with far fewer assumptions than the usual classical procedures.
- They are less sensitive to departures from the assumptions - with tests, the level is maintained even if the assumed model is incorrect. Procedures are less sensitive to extreme or unusual observations.

More Reasons

- Simplicity and intuitive nature of the procedures.
- Applicable to specific types of data for which classical procedures are not appropriate. E.g., rank and count data.
- Technology advances have increased accessibility for the general practitioner. (bootstrapping, jackknifing, simulation, etc.)

The Distribution-Free Property

- Many nonparametric tests, intervals, statistics, confidence bands, and multiple comparisons procedures are distribution-free. Roughly speaking, this means that these methods do NOT depend on the specific distributional form of the population.

General Description of One-Sample Location Problem

- Paired Replicates Data - We have an experiment such that each time it is conducted we obtain two observations. We refer to these as "pre-treatment" (X) and "post-treatment" (Y) observations. We are interested in assessing the effect of the treatment on our measured observations.
- We are developing a nonparametric competitor to the paired t-test.

Model

- Let refer to the "typical effect" of the treatment.
- Let $Z_i=Y_i-X_i$, for i=1,…, n and assume that:
  - The n Z_i's are mutually independent;
  - Each Z_i has a continuous distribution with median .
- Note that we do NOT need to assume that the Z_i's are normally distributed. Also, we do not need to assume that the Z_i's have the same distribution.

We want to make inferences for

- Three ways to use the information in the Z_i's
  - Use all of the information (Paired t-test)
  - Use the information on Y beating X and the relative magnitudes of the Z_i's. (signed rank)

–Use “minimal” information on whether or not Y beats X. (sign)

Sign Test

- We want to test

– versus

- Test Stat:

- Example

–Privgov.mtw

–SignTest.doc

Large Sample Approximation

- Since the distribution of B under the null hypothesis is Binomial(n, 1/2), the approximate level test is based on the standardized statistic

Wilcoxon Signed Rank

- Wilcoxon signed rank test statistic

where R_i is the rank of $|Z_i|$ among $|Z_1|, \ldots, |Z_n|$.

- Find the null distribution for T^+ when n=4
- Large sample approximation for the signed rank test

Another Example

- An experiment was conducted to test a method for reducing faults on telephone lines. Fourteen matched pairs of areas were used. Do the data provide significant evidence to conclude that the faults have been reduced?

Fault Data

Two-sample location problem

- Wilcoxon rank sum test statistic

–W=sum of the joint ranks for the Y-sample

- Mann-Whitney statistic

–U=the number of pairs where Y is larger than X

- Obtaining the null distribution for W
- Large sample approximation for the Wilcoxon rank sum test

Trauma and Metabolic Expenditure

- Metabolic expenditures (kcal/kg/day) were collected for 8 patients admitted to a hospital for reasons other than trauma and for 7 patients admitted for multiple fractures (trauma). Are the expenditures significantly different for the two groups of patients?

Patient Data

Point and Interval Estimation

- Form the mn differences, $Y_j - X_i$; i=1,…, m; j=1,…n.
- The point estimator for the location parameter is the median of these differences.

- Another common parameter of interest in this two sample location problem is P(Y>X). How would you estimate this probability?

Conclusion

- The more practice your students get with statistical inference, the more comfortable they will be in making appropriate inferences.
- Including alternative procedures for inference clearly illustrates why it is so important to check all assumptions and conditions.

Nonparametric Statistics: A Quick Overview

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One-sample location problem

- Sign statistic (based on counts)
- Signed rank statistic (based on counts and ranks)
- Point and interval estimation

Two-sample location problem

- Wilcoxon rank sum statistic (based on ranks)
- Mann-Whitney statistic (based on counts)
- Point and interval estimation
- Fligner-Policello statistic (based on placements)

Two-sample dispersion problem

- Ansari-Bradley statistic (based on scores)
- Miller jackknife statistic (computationally intensive, but very useful)
- Kolmogorov-Smirnov statistic (general differences in two populations)

One-way layout (k samples)

- Kruskal-Wallis statistic (general alternatives)
- Jonckheere-Terpstra statistic (ordered alternatives)

Block design (Two-way layout with one treatment factor and one blocking factor)

- Friedman statistic (general alternatives)
- Page statistic (ordered alternatives)

The Independence Problem

- Kendall statistic (based on concordant and discordant pairs)
- Spearman statistic (based on joint ranks)

Bootstrapping (used to solve a wide variety of statistical problems)

- Jerome Friedman, Stanford University (1988), "… the most important new idea in statistics in the last 20 years, and probably in the last 50. Eventually, it will take over the field of statistics."
- Persi Diaconis, Harvard University (1991), "I think the bootstrap is one of the most important developments [in statistics] of the last 10 years."

References

- Hollander, M. and Wolfe, D. A. (1998), *Nonparametric Statistical Methods*, Second Edition, New York, NY: John Wiley & Sons, Inc.
- Hettmansperger, T. P. and McKean, J. W. (1998), *Robust Nonparametric Statistical Methods*, New York, NY: John Wiley & Sons, Inc.
- Lehmann, E. L. (1975), *Nonparametrics: Statistical Methods Based on Ranks*, San Francisco, CA: Holden-Day, Inc.

Comparing Three Different Methods for Rounding First Base

Player	Roundout	Narrow Angle	Wide Angle
1	5.4	5.5	5.55
2	5.85	5.7	5.75
3	5.2	5.6	5.5
4	5.55	5.5	5.4
5	5.9	5.85	5.7
6	5.45	5.55	5.6
7	5.4	5.4	5.35
8	5.45	5.5	5.35
9	5.25	5.15	5.0
10	5.85	5.8	5.7
11	5.25	5.2	5.1
12	5.65	5.55	5.45
13	5.6	5.35	5.45
14	5.05	5.0	4.95
15	5.5	5.5	5.40
16	5.45	5.55	5.50
17	5.55	5.55	5.35
18	5.45	5.5	5.55
19	5.5	5.45	5.25
20	5.65	5.6	5.40
21	5.7	5.65	5.55
22	6.3	6.3	6.25

One Sample Sign Procedures Using Minitab for Windows

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Comparing Annual Salaries from the Private Sector with Government Salaries (Table 2)				
Pair	Priv(Yi)	Gov(Xi)	Zi	Psi_i
1	12500	11750	750	1
2	22300	20900	1400	1
3	14500	14800	-300	0
4	32300	29900	2400	1
5	20800	21500	-700	0
6	19200	18400	800	1
7	15800	14500	1300	1
8	17500	17900	-400	0
9	23300	21400	1900	1
10	42100	43200	-1100	0
11	16800	15200	1600	1
12	14500	14200	300	1

Applying the sign test with Mintab:

Select Stat > Nonparametrics > 1 Sample Sign, click Test Median, and enter q_0

Sign Test for Median

```
Sign test of median = 0.00000 versus  >  0.00000

               N  Below  Equal  Above         P     Median
Zi            12      4      0      8    0.1938      775.0
```

Computing the p-value of the sign test with Minitab:

Select Calc > Probability Distributions > Binomial, click cdf, enter n, p, and x

Cumulative Distribution Function

```
Binomial with n = 12 and p = 0.500000
         x     P( X <= x)
      7.00        0.8062
```

Computing Confidence Intervals for q with Mintab:

Select Stat > Nonparametrics > 1 Sample Sign, click CI, and enter desired level

Sign Confidence Interval

```
Sign confidence interval for median
                               Achieved
              N   Median     Confidence    Confidence interval   Position
Zi           12      775        0.8540      (     -300,    1400)        4
                                0.9500      (     -374,    1547)      NLI
                                0.9614      (     -400,    1600)        3
```

A Snapshot of the AP Statistics Program

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Major Content Areas in AP Statistics

- Exploring Data
- Design and Sampling
- Probability and Simulation
- Statistical Inference

–The revised topic outline will appear soon on AP Central.

Growth in AP Calculus and
AP Statistics

Growth Rates in AP Statistics

A Closer Look at the Numbers

YEAR	EXAMS	% PASSING (3 or higher)
1997	7,500	62.2
1998	15,500	59.7
1999	25,000	57.1
2000	37,000	53.7
2001	42,000	59.7
2002	49,900	56.8
2003	58,200	61.9
2004	66,000	59.8
2005	?	?

Score Distributions
for 2003 and 2004

Score	2003 Percent	2004 Percent
5	13.2	12.5
4	22.3	22.5
3	26.4	24.8
2	19.5	19.8
1	18.6	20.4

Student Performance
(over 5 years)

Highlights from 2004 Reading

- Overall performance on the multiple choice questions was better than the previous two years.
- The scores on the free response questions were lower than the previous two years.
- The overall average was down slightly, but very close to the averages in 2002 and 2003.

The Best and Worst of 2004

- The best news is that all six questions showed good discrimination across the entire range of scores.

- The most discouraging news is that students continued to perform poorly on standard problems dealing with statistical inference.

Common Student Errors
(2004 Edition)

- Many students provided solutions with no justification or an incomplete justification. Students need to be encouraged to show all of their work and justify their answers.

- Students failed to read the problems carefully and then provided information that was not relevant to the questions that were asked.

- Communication of statistical analyses and concepts continues to be a problem.

Scoring Statistics

Question	2003	2004
1	1.97	1.34
2	1.62	1.59
3	1.84	1.39
4	1.69	1.37
5	1.35	1.35
6	1.40	1.25

College Comparability Study

- College students from selective to highly selective colleges and universities across the country completed free response questions 1, 5, and 6.

- Consider the logistical hurdles that must be dealt with when trying to complete this type of study.
  – Contact time, content of the course, text, quality of students, vocabulary, format of the test, etc.

Institutions Participating in the College Comparability Study
Baylor University
California Polytechnic State University
Columbia University
George Washington University
Oberlin College
Ohio State University
Texas A&M
The College of New Jersey
University of California-Davis
University of Wisconsin-Milwaukee

Results of the College Comparability Study

- AP Statistics students did much better on the exam than college students
  – The averages for the college students on all three free response questions (1, 5, and 6) were substantially below the corresponding averages for the AP students.

–The multiple choice scores were better for AP students.

- In short, AP students stand out in every comparison that was made with college students.
- This is very encouraging and exciting news, not only for the AP Statistics Program, but for everyone involved with statistics education.

Recommendations for Teachers

- Emphasize conceptual understanding and communication over mechanics.
- Be sure that students are comfortable reading output from statistical software – integrating computer use into your course is preferable.
- Emphasize the importance of **checking** conditions for statistical procedures.
- Ask your students to answer questions in context.

More Suggestions

- Give your students time to discuss and debate their solutions in a group setting.
- Provide as many practical applications and problems as possible. Students need to practice applying the concepts they have learned before they get to the AP Exam.
- Writing assignments are essential for success – communication continues to be an area of weakness for many students.

A Concern of the AP Statistics Test Development Committee

- Over several years there has been considerable variability in the scores on questions covering content area II.
- The committee recommends that workshop leaders spend a substantial amount of time on this topic.
- The committee also recommends that teachers cover this material early in the course and return to these ideas frequently.
- If at all possible, students should plan and conduct their own study.

Question and Answer Session

- Curricular Issues
- Course Content
- Statistical Software
- Course Projects
- Post-Exam Activities
- Test Development
- The Exam
- Rubrics
- The Reading
- Grade Setting