

September 2009 Puzzle

Ryan and Alyssa's piggy banks contain exactly the same amount of money consisting of pennies, nickels, dimes, and quarters. While they both have the same amount of money, Alyssa has four more quarters than Ryan does, however Ryan has more of each of the other coins. In total, Ryan has 32 more coins than Alyssa. Given that Alyssa has 23 nickels, how many nickels does Ryan have?

September 2009 Solution

The answer to the puzzle is that **Ryan has 30 Nickels.**

In the table to the right, we define several variables to represent our unknowns and also include our knowledge that Alyssa has 23 nickels and 4 more quarters than Ryan.

	Pennies	Nickels	Dimes	Quarters
Alyssa	P_A	23	D_A	$Q + 4$
Ryan	P_R	N_R	D_R	Q

The fact that Ryan and Alyssa have the exact same amount of money can be expressed as follows (in cents):

$$1P_A + 5(23) + 10D_A + 25(Q + 4) = 1P_R + 5N_R + 10D_R + 25Q$$

After simplifying the equation by combining like terms we obtain

$$P_R - P_A + 5N_R + 10(D_R - D_A) = 215$$



We now express the fact that Ryan has in total, 32 more coins than Alyssa, as follows:

$$P_R + N_R + D_R + Q = P_A + 23 + D_A + (Q + 4) + 32$$

Manipulating the equation gives us

$$P_R - P_A = D_A - D_R - N_R + 59$$

Substituting for $P_R - P_A$ in the equation given in ★ above yields

$$(D_A - D_R - N_R + 59) + 5N_R + 10(D_R - D_A) = 215$$

We now solve for N_R to obtain

$$N_R = 39 - \frac{9(D_R - D_A)}{4}$$

Since N_R represents the number of nickels that Ryan has, it must be a whole number and therefore the term $\frac{9(D_R - D_A)}{4}$ has to be a whole number as well. This requires $D_R - D_A$ to be a positive multiple of 4; positive because Ryan has more dimes than Alyssa. We also cannot have $D_R - D_A$ a multiple of 4 that is 8 or higher as that would result in the Ryan having fewer than the 23 nickels that Alyssa has, which again we know isn't the case. Thus the only possible solution corresponds to $D_R - D_A = 4$, which by the equation above yields $N_R = 30$, i.e. Ryan has 30 nickels.