**February 2009 Puzzle**

Fill in the following grid as much as possible using A’s, B’s, C’s, D’s, E’s, and F’s so that no row, column, or diagonal contains any letter more than once. It is impossible to fill the grid completely this way, but it is your job to leave as few blanks as possible.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
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**February 2009 Solution**

Since there are 6 rows (and columns), the maximum number of times that any one letter can occur in the grid is 6. With little effort one can see that it is impossible to arrange 6 A’s or 6 F’s with the given restrictions. In addition, one finds that for each of the letters B, C, D, and E, there is only one way to arrange them with 6 occurrences. These arrangements are shown below.

![Arrangements A, B, C, D, E, F](image)

Now, we already know that the completed grid must contain at least 2 blanks as we said it is impossible to include 6 A’s or 6 F’s. We might first try to minimize the number of blanks by merging each of the four arrangements shown above in hopes of completing the grid with just 2 blanks. Below you will find this merged arrangement.

![Merged Arrangement](image)

With this arrangement, we would only be able to include one additional A and one additional F, as only the main (longest) diagonals have blanks remaining and each already contains either an A or F (in the top row). That would leave 8 blanks; we can do better than that.

Our next best attempt would be to try to merge three of the four arrangements above. Regardless of which three we merge, we will be confronted with the same issue. Let us for example consider the merging of the first three.

![Merged Arrangement 3](image)

We are left to insert the A’s, E’s and F’s. There are 15 blanks spaces remaining, 10 of which are on the main diagonals. Because each of the main diagonals already contains an A or F, we can place at most one more A and one more F on the main diagonals. In addition, we could place at most one E on each of the diagonals. This would leave us with 6 blanks just on the main diagonals. So in this merging as in any merging with three of the four arrangements above, we could hope for, at best, 6 blanks at the end.

It turns out that merging (any) two of the four arrangements above will lead to an optimal solution consisting of just 4 blanks. There are many different solutions; we will just provide just one (see the grid to the right) which has 6 C’s, 6 D’s, and 5 of each of the other letters. Based on the above arguments, we could do no better.