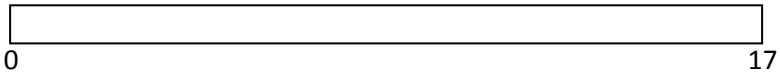
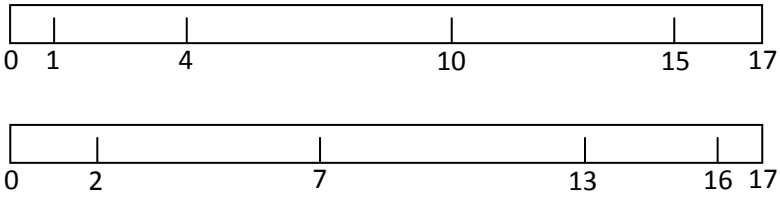
September 2008 Puzzle

Add four distinct (vertical) marks to the following ruler of length 17 so that no smaller length can be measured more than once using this ruler. Your ruler will not be able to measure all lengths up to 17 (it will be able to measure 15 distinct nonzero lengths), but an additional requirement will be that you must construct the ruler so that it can be used to measure lengths 5, 10, and 15 as well as twelve others.



September 2008 Solution

There are two possible solutions to this puzzler although mathematicians might consider them equivalent as one is just the "reverse" of the other. Here they are:



Both rulers are obtained by placing the marks 1-, 4-, 10-, and 15- units from one of the ends of the rulers; the left end in the first ruler and the right end in the second ruler. We will use the first ruler to describe how the fifteen desired non-zero lengths can be measured:

0 to 1: 1 unit	0 to 4: 4 units	1 to 4: 3 units	0 to 10: 10 units	1 to 10: 9 units
4 to 10: 6 units	0 to 15: 15 units	1 to 15: 14 units	4 to 15: 11 units	10 to 15: 5 units
0 to 17: 17 units	1 to 17: 16 units	4 to 17: 13 units	10 to 17: 7 units	15 to 17: 2 units

One can observe that we measured all (integer) lengths up to 17 with the exception of 8 and 12. Showing that there are no other solutions is a tedious process, which we have chosen to omit from this solution.