

April 2008 Puzzle

Cattle are placed into a pasture containing a given amount of grass per acre, and the grass continues to grow at a constant rate. If 10 cattle can consume the grass on 8 acres in 15 weeks and if 15 cattle can consume the grass on 8 acres in 9 weeks, how many cattle would it take to consume the grass on 64 acres in 6 weeks?

We assume that the cattle all eat at the same constant rate.

April 2008 Solution

An important thing to realize in this puzzle is that the grass is constantly growing as the cattle eat it. So eating 8 acres of grass over a course of 15 weeks requires the consumption of more grass than eating 8 acres over the course of 9 weeks. We are however given that the cattle will consume the grass at a constant rate (volume per unit of time) and that the grass grows at a constant rate. Let us now introduce a few variables to work with.

Let G = amount of grass per acre present a time 0, when the cattle begin eating. (Units for G : cubic feet)

Let R = the constant rate at which the grass grows on each acre is. (Units for R : cubic feet/week)

Let T = the amount of time elapsed since the cows began eating. (Units for T : weeks)

So the volume of grass available per acre from time 0 to time T , will be $G + TR$.

If 10 cattle eat the grass on 8 acres in 15 weeks, then the 10 cattle will eat a total volume of $8(G + 15R)$ cubic feet of grass during those 15 weeks. Therefore the 10 cattle will eat $\frac{8(G+15R)}{15}$ cubic feet of grass per week, and thus each of the cattle will eat $\frac{8(G+15R)}{150}$ cubic feet of grass per week. Of course we don't know what this rate is, because we were not told what the values of G and R are. However, we were told that the cattle eat at a constant rate. Thus if we repeat this analysis for the second scenario, we obtain a second equivalent formula for this rate.

The second scenario indicates that 15 cattle can eat the grass on 8 acres in 9 weeks. This means that the 15 cattle will eat a total volume of $8(G + 9R)$ cubic feet of grass during those 9 weeks. From this we see that the 15 cattle will eat $\frac{8(G+9R)}{9}$ cubic feet of grass per week and that each of the cattle will eat $\frac{8(G+9R)}{135}$ cubic feet of grass per week. Since the cattle eat the grass at a constant rate we must have that

$$\frac{8(G+15R)}{150} = \frac{8(G+9R)}{135}$$

Solving this equation for G yields $G = 45R$. Substituting this into the expression on the left-side of the equation above tells us that the cattle consume grass at a constant rate of $\frac{8(G+15R)}{150} = \frac{8(45R+15R)}{150} = \frac{16R}{5}$

Our third and final scenario has an unknown number X of cattle eating the grass on 64 acres in 6 weeks. Like above we determine that the cattle are eating at a rate of $\frac{64(G+6R)}{6X} = \frac{32(45R+6R)}{3X} = \frac{544R}{X}$. But this rate must be the same as that obtained above. That is, we must have $\frac{544R}{X} = \frac{16R}{5}$. Solving for X yields $X =$ **170 cattle**.